Magnetic Deviation on a vessel
and what to do about it.
by: Richard R. Shiffman
Digital Graphics Assoc.
rrs@rickshiffman.net

There are several effects on a boat that cause the magnetic compass not to point to true north. True north and south poles of the Earth are located on the planet's rotational axis. The first problem is that the Earth's magnetic poles are not aligned with the rotational axis of the Earth. The magnetic north pole at the present time is somewhere in northern Canada and the Earth's magnetic field is not exactly a dipole. Because of this, when you are in Los Angeles your magnetic compass points approximately 14 degrees east of the geographic or the true north pole. If you are around New York the magnetic north pole is approximately 9 degrees west of the geographic north pole.

This difference between true north and magnetic north is called variation and it depends on where you are on the surface of the Earth. To change between magnetic course and true course you must add easterly variation, and subtract westerly variation. Variation must be looked up on the local nautical chart. Your boat can't do anything about it.

In general, to correct from a compass course to a true course you must add easterly compass errors and subtract westerly ones. Deviation will be discussed a little later in this paper.

E is easterly differences
W is westerly differences

\[
\begin{align*}
E & := 1 \\
W & := -1
\end{align*}
\]

to change from magnetic to true or visa-versa use the following formulas

\[
\begin{align*}
\text{TrueC} = & \text{MagC} + \text{dov} \cdot E \\
\text{TrueC} = & \text{MagC} + \text{dov} \cdot W \\
\text{MagC} = & \text{TrueC} - \text{dov} \cdot E \\
\text{MagC} = & \text{TrueC} - \text{dov} \cdot W
\end{align*}
\]

Where TrueC is the true course and MagC is the magnetic course. dov is the absolute value of the degrees of variation and E or W indicates whether its easterly or westerly direction.

Now that we understand how to convert from magnetic to true courses and back again from true to magnetic, we must master converting from magnetic course to compass course and back again. This will allow us to compute what compass course we have to steer if we want to move on a true course plotted on the nautical chart or to compute the true course that we are on when steering a compass course. Note: Sailors have been doing this for many years, even before Mathcad. So it isn't very hard, and they have several politically incorrect sayings to remember these formulas.
Now we are finally getting to deviation. It's the difference between your magnetic heading and what your steering compass reads. It is caused by the vector sum of the Earth's magnetic field, which is your magnetic heading, and the boat's magnetic field. This is different for each boat and must be measured.

Some of the causes of the boat's field are the engine block, if it's iron and the DC electrical system if twisted pair wiring isn't used. Also, any large ferrous metal object fairly near the steering compass. The electrical system on the boat can lead to changes in the magnitude and direction of the boat's field as different pieces of equipment, like the radar, are powered up and down.

E is easterly differences $E = 1$

W is westerly differences $W = -1$

to change from compass to magnetic or visa-versa use the following formulas

$$\text{MagC} = \text{CompC} + \text{dod} \cdot E$$

$$\text{MagC} = \text{CompC} + \text{dod} \cdot W$$

$$\text{CompC} = \text{MagC} - \text{dod} \cdot E$$

$$\text{CompC} = \text{MagC} - \text{dod} \cdot W$$

Where MagC is the magnetic course and CompC is the compass course. dod is the absolute value of the degrees of deviation and E or W indicates whether it's easterly or westerly direction. As with variation, when you are correcting add easterly deviations and subtract westerly deviations.

Now let's examine an example of deviation for the hypothetical case where the boat's field and the Earth's magnetic field are of the same magnitude. Also the direction of the Earth's field is fixed at 000 degrees and the boat's field is free to rotate with the boat. In these examples the boat field is aligned with the fore-aft axis of the boat. The magnetic field at the compass will be the vector sum of the boat's field and the Earth's field.

The Earth's field

$$\text{EF}(\alpha) := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The Boat's field relative to the Earth

$$\text{BF}(\alpha) := \begin{bmatrix} \cos \left( \frac{2 \pi \alpha}{360} \right) & -\sin \left( \frac{2 \pi \alpha}{360} \right) \\ \sin \left( \frac{2 \pi \alpha}{360} \right) & \cos \left( \frac{2 \pi \alpha}{360} \right) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The field at the Compass

$$\text{CF}(\alpha) := \text{EF}(\alpha) + \text{BF}(\alpha)$$

Setup to plot the Earth's magnetic field, the hypothetical boat's field, and the resultant field at the compass. You can easily see the effect of deviation on the boat's compass.

$$\text{angle}(a) := \text{if}(a = 0, 10^{-6}, a)$$

$$i := 0..10 \quad d\tau := .1 \quad di := 1 \quad \tau := \frac{d\tau}{di}\cdot i$$

$$\text{angV}(v0, v1) := \text{acos} \left( \frac{v0 \cdot v1}{|v0| \cdot |v1|} \right) \cdot \frac{360}{2\pi}$$

Calculate the angle between 2 vectors in degrees
The following plots use the standard mathematical form, not a boat compass display. An angle of 000 degrees is on the positive x axis, 090 degrees is on the positive y axis, 180 degrees is aligned with the negative x axis, and 270 degrees is aligned with the negative y axis. An easterly movement is represented by a counterclockwise rotation. Westerly movement would be a clockwise rotation.

In the first example the boat is on a magnetic course of 045 degrees. You can try other angles and see what effect that has on the plot below. Remember, by definition the angle of the Earth’s field is 000 and every angle is relative to that.

\[ a_{\text{EF}} := \arg(\text{EF}(\alpha)_0 + i \cdot \text{EF}(\alpha)_1) \]  
Calculate the angle of the Earth’s field.

\[ a_{\text{BF}} := \arg(\text{BF}(\alpha)_0 + i \cdot \text{BF}(\alpha)_1) \]  
Calculate the angle of the Boat’s field.

\[ a_{\text{CF}} := \arg(\text{CF}(\alpha)_0 + i \cdot \text{CF}(\alpha)_1) \]  
Calculate the angle of the field at the compass.

Calculate vectors to plot

\[ \text{PEF}_i := \text{EF}(\alpha) \cdot \tau_i \]  
\[ \text{PBF}_i := \text{BF}(\alpha) \cdot \tau_i \]  
\[ \text{PCF}_i := (\text{EF}(\alpha) + \text{BF}(\alpha)) \cdot \tau_i \]

Compass Field plot

- Earth’s Field
- Boat’s Field
- Compass’ Field

\[ \text{Deviation} \left( \frac{a_{\text{BF}}}{\text{deg}}, \frac{a_{\text{CF}}}{\text{deg}} \right) = 22.5 \text{ deg east} \]
In the second example the boat is on magnetic course of 315 degrees. In this case the deviation is westerly.

Again, 000 degrees is aligned with the positive x axis, and easterly movement is represented by a counterclockwise rotation. Westerly movement would be a clockwise rotation.

\[ a_{EF} := \arg(\{\text{EF}(\alpha)\}_0 + i \cdot \text{EF}(\alpha)_1) \] Calculate the angle of the Earth's field.

\[ a_{BF} := \arg(\{\text{BF}(\alpha)\}_0 + i \cdot \text{BF}(\alpha)_1) \] Calculate the angle of the Boat's field.

\[ a_{CF} := \arg(\{\text{CF}(\alpha)\}_0 + i \cdot \text{CF}(\alpha)_1) \] Calculate the angle of the field at the compass.

\[ P_{EF} := \text{EF}(\alpha) \cdot \tau_i \quad P_{BF} := \text{BF}(\alpha) \cdot \tau_i \quad P_{CF} := (\text{EF}(\alpha) + \text{BF}(\alpha)) \cdot \tau_i \]

\[ \text{aEF} = 0 \cdot \text{deg} \quad \text{Angle of Earth's Field} \]
\[ \text{aBF} = -45 \cdot \text{deg} \quad \text{Angle of boat's Field} \]
\[ \text{aCF} = -22.5 \cdot \text{deg} \quad \text{Angle of compass field} \]

\[ E = 1 \quad \text{positive is easterly deviation} \]
\[ W = -1 \quad \text{negative is westerly deviation} \]

Again we use \( \text{Deviation}(m,c) \) to calculate the deviation in degrees,

\[ \text{Deviation} \left( \frac{\text{aBF}}{\text{deg}}, \frac{\text{aCF}}{\text{deg}} \right) = -22.5 \quad \text{deg west} \]

As you can see, a deviation table for each boat must be determined. This table should list the magnetic heading and the deviations for the cardinal and sub-cardinal heading. This would be the minimal deviation table, a more complete table would list the deviation for every 15 degrees of the compass card. If the boat has large deviations (more than 5 degrees in magnitude) than a table of compass course and deviation is also helpful. These tables provide you with the information needed to convert between magnetic course and compass course for any heading of the boat.
A couple of methods for determining your deviation table will now be discussed. One method is to use a hand bearing-compass and stand at least four feet or more from ferrous objects and electrical wires on the boat. You can now assume that the hand bearing-compass is deviation free and reads the vessel's magnetic course. Make sure that you verify that the hand bearing-compass is deviation free before continuing. Then sight parallel to the steering compass' lubber's line with the hand bearing-compass, use the hand bearing-compass to measure the magnetic heading of the boat when the boat is on the compass course of a table entry. If you can take multiple readings for each compass course, then average the magnetic bearings to get a better table entry. The author used this technique to produce the table at the end of this paper.

The next technique is the classical method using a range and a pelorus, an instrument used to measure the relative bearing from the bow of the boat. When you cross a range, at that moment you are on a line of position of a known true direction. From C,D,M,V,T add east (Compass, Deviation, Magnetic, Variation, True), where deviation and variation are compass corrections, you can easily calculate the magnetic bearing of the range. From the magnetic bearing of the range and the Pelorus you can arrange to cross the range at known magnetic headings. When you cross the range at a known magnetic heading, recorded the compass course from the steering compass, enabling you to calculate a deviation table entry. Repeating this for other magnetic headings allows you to build up the deviation table for your boat.

A third way is to use your GPS or gyrocompass if you have one. As to gyrocompasses, most small boats that the author is familiar with, don't have one. If you do, just use it to read off the magnetic heading and subtract the compass heading from it to find the deviation.

Many small boats now are equipped with a GPS receiver. With some care you can use it with your steering compass to generate a deviation table. The true or magnetic boat heading that your GPS displays has nothing to do with the direction that the boat's pointing. It is where the boat's velocity vector is pointing. If your propulsion is astern with the bow pointing due south, the GPS will state that you are on a 000 degree true north course! Also the GPS read velocity over the ground, not velocity through the water. Thus your velocity over ground (locally) will be the vector sum of your velocity through the water and the set and drift of the current. Note: the microprocessor in all GPS's is smart enough to handle variation anywhere in the world, and convert between true and magnetic heading for you. Taking the above into account and traveling with a great enough velocity to minimize the effect of current, you can use your GPS's magnetic heading minus the steering compass bearing to calculate your deviation table entries.

Now on to the calculations to produce the deviation table for my boat. The data used for this table was taken on 05-03-98.
New deviation table for my boat, Bigger Dinghy ...

Data taken on 05-03-98 at 1421T

We will calculate the deviation table for the 8 compass courses listed below

\[
\text{ic} := 0 .. 7 \quad \text{leftc}_{\text{ic}} := 045 \cdot \text{ic}
\]

"leftc" is a vector of compass courses for which the deviation is evaluated at. We will need to compare them to the magnetic courses. The difference of the magnetic course and the compass course is the compass deviation on that course.

\[
\text{leftc}^T = \begin{bmatrix} 0 & 45 & 90 & 135 & 180 & 225 & 270 & 315 \end{bmatrix}
\]

\[
\text{aveDir} \left( \text{dirs} \right) := \begin{cases} 1 & \text{length(} \text{dirs} \text{)} \text{ for } i \in 0 .. 1 \end{cases}
\]

\[
x_{\text{dirs}}_i \leftarrow \cos \left( \frac{2 \cdot \pi}{360} \cdot \text{dirs}_i \right)
\]

\[
y_{\text{dirs}}_i \leftarrow \sin \left( \frac{2 \cdot \pi}{360} \cdot \text{dirs}_i \right)
\]

\[
x \leftarrow \frac{\sum x_{\text{dirs}}}{1}
\]

\[
y \leftarrow \frac{\sum y_{\text{dirs}}}{1}
\]

\[
a \leftarrow \arg \left( x + y \cdot \sqrt{-1} \right) \cdot \frac{360}{2 \cdot \pi}
\]

\[
a < 0 \quad \text{if} \quad \langle a \leftarrow 360 + a \rangle \mod \langle a, 360 \rangle
\]

Test to make sure that 001 degrees and 361 degrees generate the same answer with aveDir.

\[
\text{aveDir} \left( \begin{bmatrix} 001 & 358 & 359 \end{bmatrix}^T \right) = 359.333 \quad \text{aveDir} \left( \begin{bmatrix} 361 & 358 & 359 \end{bmatrix}^T \right) = 359.333
\]

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\[
\text{rndoff}(x,f) \text{ is used to round numbers off. } x \text{ is the number to round and } f \text{ is the decimal fraction to round off to. If you want one decimal place set } f := 0.1. \text{ For two places set } f := 0.01. \text{ Rnd}(x,n) \text{ rounds } x \text{ off to the } n\text{'th decimal place.}
\]

\[
\text{rndoff}(x,f) := \left\lfloor \left( \frac{x + 0.5 \cdot f}{f} \right) \cdot \frac{1}{f} \right\rfloor
\]

\[
f(p) := \frac{1}{10^{\left\lfloor \text{floor}[p] \right\rfloor}}
\]

\[
\text{Rnd}(x, n) := \text{rndoff} \left( \left| x \right|, f(n) \right) \cdot \frac{x}{\left| x \right|}
\]
Below are the compass courses and the corresponding measured magnetic heading data. The measured data is highlighted.

<table>
<thead>
<tr>
<th>Compass Course</th>
<th>Magnetic Headings</th>
</tr>
</thead>
<tbody>
<tr>
<td>leftc₀ = 0</td>
<td>vect₀ := [001 358 359]^T</td>
</tr>
<tr>
<td>leftc₁ = 45</td>
<td>vect₁ := [043 042 044]^T</td>
</tr>
<tr>
<td>leftc₂ = 90</td>
<td>vect₂ := [087 088 089]^T</td>
</tr>
<tr>
<td>leftc₃ = 135</td>
<td>vect₃ := [130 132 135]^T</td>
</tr>
<tr>
<td>leftc₄ = 180</td>
<td>vect₄ := [177 177 176]^T</td>
</tr>
<tr>
<td>leftc₅ = 225</td>
<td>vect₅ := [219 217 218 222 223]^T</td>
</tr>
<tr>
<td>leftc₆ = 270</td>
<td>vect₆ := [269 270 269]^T</td>
</tr>
<tr>
<td>leftc₇ = 315</td>
<td>vect₇ := [312 316 314]^T</td>
</tr>
</tbody>
</table>

Calculate the average magnetic headings and then the deviation table. The results are rounded to the nearest degree.

\[
\text{Rnd}0\text{aveDir}(\text{vect}) := \text{Rnd}\left(\text{aveDir}(\text{vect}), 0\right) \quad \text{Deviation}(m, c) \equiv \begin{cases} 
\text{dev} \leftarrow \text{mod}(m - c, 360) \\
\text{dev} \leftarrow \text{dev} - 360 \quad \text{if} \quad \text{dev} \geq 0 \\
\text{dev} \leftarrow 360 + \text{dev} \quad \text{if} \quad \text{dev} < -180
\end{cases}
\]

\[\text{mag}_{ic} := \text{Rnd}0\text{aveDir}\left(\text{vect}_{ic}\right)\]

\[\text{Dev}_{ic} := \text{Deviation}\left(\text{mag}_{ic}, \text{leftc}_{ic}\right)\]

Above is the global definition of Deviation(m,c) function. All inputs and output are in degrees.

Below is the Deviation Table for the left compass of my boat.

<table>
<thead>
<tr>
<th>leftc₀</th>
<th>mag₀</th>
<th>Dev₀</th>
<th>Negative deviations are westerly and positive deviations are easterly. This is the current deviation table for my left compass as of 980503.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>359</td>
<td>-1</td>
<td>E = 1 \quad W = -1</td>
</tr>
<tr>
<td>45</td>
<td>-43</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>-88</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>132</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>177</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>225</td>
<td>220</td>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>269</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>315</td>
<td>314</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>
References