Phil. 2440
Course Requirements, What is logic?

To discuss today:

- *About the class:*
  - Some general course information
  - Who should take this class?
  - Course mechanics
  - What you need to do
- *About logic:*
  - Why is it important?

About the course:

Some general course information

- Professor: Michael Huemer <owl1@free-market.net>
- Office hours: MWF, 1-2, in Prufrock's.
- Web page: http://home.sprynet.com/~owl1/244.htm

Subject matter of the course:

- Propositional logic
- Predicate logic
- Set theory
- Metalogic + Gödel's Theorem

Course requirements:

- Tests.
- Homework problems. Guidelines (see syllabus):
  - May discuss, do not copy
  - Lateness: 2/3 credit
  - Sending by email
  - Grading

Miscellaneous guidelines for the course:

- Come on time.
- Come to office hours.
- Question.
- Grading: the curve:
  - (Adjusted grade) = (Raw score)(n) + 100(1- n)

What do you need to do now?

- Get the course reader.
- Read the syllabus.
- Read chapter 1.
- For Friday: do questions on chapter 1

About logic:

Why is logic important for philosophers?

- The importance of arguments in philosophy
- Logic teaches us about the structure of propositions.
  - Many philosophical theses/issues could not be formulated without modern, formal logic.
  - You should be able to understand modern philosophers.
- Can logic help us make progress in philosophy?
  - To think about: how did modern science make progress?
  - The role of mathematics in modern science.

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Chapter 1: General Introduction

To discuss today:
- What logic is
- Arguments
- Basic concepts used in logic
- Some silly-sounding principles of logic

The subject matter of logic:
- What is logic?
- What is reasoning?
- The importance of reasoning
- 'Correct' vs. 'incorrect' reasoning. Two kinds of mistakes:
  - False premises
  - Invalid reasoning
- Logical vs. psychological questions.
  - Logical: is this a good argument for that?
  - Psychological: why do people believe this?

About Arguments:
- What are they?
- Premises & conclusions
- Validity and soundness
  - 'Valid' arguments: It is impossible that the premises all be true and the conclusion be false.
  - 'Sound' arguments: Valid + all true premises
- Deductive, inductive, and other arguments
  - Deductive: purports to be valid
  - Non-deductive: purports to support conclusion but not to be valid. Renders conclusion more probable.
  - Inductive: example: "All ravens so far observed have been black. So (probably) all ravens are black.

Important logical concepts & distinctions
- Statements vs. sentences
- Statements, beliefs, and propositions
- What is truth?
  - Aristotle: "To say of what is, that it is, is true."

- Logical possibility--the received view. Which of the following are possible?
    - “The solar system has nine planets.”
    - “The solar system has 62 planets.”
    - “My cat wins the world chess championship next year.”
    - “The law of conservation of mass/energy is false.”
    - “My car is completely red and completely green at the same time.”
    - “Sam is a married bachelor.”
    - “2 + 2 = 7.”
    - “It is raining and it is not raining.”
    - “Some things are neither red nor not red.”
- What is wrong with the received view
- Other senses of ‘possible’
    - Epistemic possibility
    - Physical possibility
    - Metaphysical possibility
- Logical truth and falsity
- Contingent propositions
- Contradictions
- Entailment
- Logical equivalence

Silly doctrines of modern logic

- Is this valid:
    - It is raining.
    - It is not raining.
    - Therefore, Skeezix is furry.
- Is this valid:
    - All men are mortal.
    - Socrates is a man.
    - Therefore, it is either raining or not raining.
- Three definitions of “valid”
    - If the premises are true, the conclusion must be true.
    - The conclusion follows from the premises.
    - It is not possible that the premises be true and the conclusion be false.

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Chapter 2: Propositional Calculus Symbolizations

To discuss today:
- Formal systems in general.
- How to symbolize things in propositional logic.
- Miscellaneous logical terminology/concepts.

About formal systems
- What's a formal system?
- What are formal systems good for?
- The propositional calculus
- Compound vs. atomic sentences

Propositional calculus symbols

symbol	what it means	example	other comments
A	Stands for any atomic sentence.	"Alice got a haircut" can be symbolized as A. "Bert owns a cat" can be symbolized as B.	You can use any capital letters, not just "A".
∨	or	"Bill has an elephant in his apartment, or he's very fat" = (E ∨ F)	This symbol is called a "vel".
&	and	"I went to the store today and I bought a cow" = (S & C)	Sometimes people use "•" or "∧" (without the quote marks) for this.
~	not	"I did not go to the store today" = ~S	This one is called a "tilde". Sometimes they use "¬".
→	If ... then ...	"If Bill Clinton was a great President, then I'm a monkey's uncle" = (G → M)	People also use "⊃".
↔	... if and only if ...	"I will go to the party if and only if you go" = (I ↔ Y)	People also use "≡".

symbol	what it means	example	other comments
( )	Parentheses are used to avoid ambiguity (see below).	"If Liz and Sue go, I will go" = ((L & S) → I) "Liz will go, and if Sue goes I will go" = (L & (S → I))	Used when you join together two other sentences with "∨", "&", "→", or "↔"

Things to notice:

- Use parentheses to avoid ambiguity.
- Inclusive 'or'.
- If and only if
- "And" in English

Other terminology

- "propositional constant"
- "connective"
- "conjunction", "conjunct"
- "disjunction", "disjunct"
- "negation", "negatum"
- "conditional", "antecedent", "consequent"
- "biconditional"

Other English connectives

- "but", "so", "although"
- "A only if B"
- "A if B"
- "provided that", "assuming"
- "unless"
- "neither ... nor"
- "not both"

Miscellaneous stuff

- Well-formed formulas
- Compound vs. atomic sentences: compound sentence contains connective(s)
- The main connective
- Propositional variables
- Forms: What are they?
- Substitution instances

What are sentence forms good for?

A sentence can have many different forms

- Example: (B ↔ (B & C))
- Forms:
  - [p ↔ (p & q)]
  - [p ↔ (q & r)]
  - (p ↔ q)
  - p

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Chapter 3: Truth Tables

To Discuss Today:
- Truth tables:
  - Defining connectives with them
  - Using them to evaluate arguments
- Limitations of propositional logic

Truth Tables for Defining Connectives

Background concepts:
- Truth values
- Functions
- Truth-functions & "truth-functional" connectives

What is a truth-table?

	p	q	(p & q)
1.	T	T	T
2.	T	F	F
3.	F	T	F
4.	F	F	F

p	~p
T	F
F	T

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

The material conditional & material equivalence:
- "If A then B" = "Not: (A and not-B)"
- Truth-table:

p	q	p → q
T	T	T
T	F	F
F	T	T

p	q	p ↔ q
T	T	T
T	F	F
F	T	F

F	F	T

F	F	T

Defining connectives in terms of other connectives

$(A \leftrightarrow B) = (A \rightarrow B) \;\&\; (B \rightarrow A)$

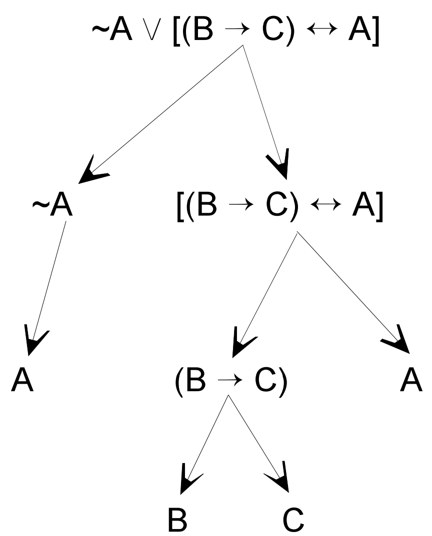
$(A \rightarrow B) = {\sim}(A \;\&\; {\sim}B)$

$(A \;\&\; B) = {\sim}({\sim}A \vee {\sim}B)$

Q: Can all the connectives be defined in terms of a single connective?

Truth-Tables for Evaluating Arguments

Breaking a complex sentence into parts



Result: A, B, C, ~A, $(B \rightarrow C)$, $[(B \rightarrow C) \leftrightarrow A]$, $\{{\sim}A \vee [(B \rightarrow C) \leftrightarrow A]\}$

Truth-tables for complex sentences

We need 2^n lines in the table, n=# of atomic sentences

Columns for each part of the sentence

Fill in T’s and F’s for atomic sentences

	A	B	C	~A	B → C	(B → C) ↔ A	~A ∨ [(B → C) ↔ A]
1.							
2.							
3.							
4.							
5.							
6.							
7.							
8.							

	A	B	C	~A	B → C	(B → C) ↔ A	~A ∨ [(B → C) ↔ A]
1.	T	T	T				
2.	T	T	F				
3.	T	F	T				
4.	T	F	F				
5.	F	T	T				
6.	F	T	F				
7.	F	F	T				
8.	F	F	F				

	A	B	C	~A	B → C	(B → C) ↔ A	~A ∨ [(B → C) ↔ A]
1.	T	T	T	F	T		
2.	T	T	F	F	F		
3.	T	F	T	F	T		
4.	T	F	F	F	T		
5.	F	T	T	T	T		
6.	F	T	F	T	F		
7.	F	F	T	T	T		
8.	F	F	F	T	T		

	A	B	C	~A	B → C	(B → C) ↔ A	~A ∨ [(B → C) ↔ A]
1.	T	T	T	F	T	T	T
2.	T	T	F	F	F	F	F
3.	T	F	T	F	T	T	T
4.	T	F	F	F	T	T	T
5.	F	T	T	T	T	F	T
6.	F	T	F	T	F	T	T
7.	F	F	T	T	T	F	T
8.	F	F	F	T	T	F	T

More compact way of doing truth tables:

Stage 1:

	A	B	C	~A ∨ [(B → C) ↔ A]
1.	T	T	T	F T
2.	T	T	F	F F
3.	T	F	T	F T
4.	T	F	F	F T
5.	F	T	T	T T
6.	F	T	F	T F
7.	F	F	T	T T
8.	F	F	F	T T

Stage 2:

	A	B	C	~A ∨ [(B → C) ↔ A]
1.	T	T	T	F T T
2.	T	T	F	F F F
3.	T	F	T	F T T
4.	T	F	F	F T T
5.	F	T	T	T T F
6.	F	T	F	T F T
7.	F	F	T	T T F
8.	F	F	F	T T F

Stage 3:

	A	B	C	~A ∨ [(B → C) ↔ A]
1.	T	T	T	F T T T
2.	T	T	F	F F F F

3.	T	F	T	F T	T	T	
4.	T	F	F	F T	T	T	
5.	F	T	T	T T	T	F	
6.	F	T	F	T T	F	T	
7.	F	F	T	T T	T	F	
8.	F	F	F	T T	T	F	

Testing validity

Is there a line in which all premises are true & conclusion is false?

Example: Is this valid?:

~(A → B)

∴ (A ∨ B)

	A	B	A → B	~(A → B)	A ∨ B
1.	T	T	T	F	T
2.	T	F	F	T	T
3.	F	T	T	F	T
4.	F	F	T	F	F

Other uses:

Identifying contradictions

Identifying tautologies

Contingent propositions

Limitations of the Propositional Calculus

The material conditional: Does it correspond to the ordinary meaning of "if...then"? Are these valid:

Example 1:

If I put sugar in my coffee, it will taste fine.

∴ If I put sugar and motor oil in my coffee, it will taste fine.

Example 2:

I have no orange juice in the refrigerator.

∴ If I have orange juice in the refrigerator, then the world will come to an end.

Example 3:

It's not the case that if God exists, the universe is the product of blind chance.

∴ God exists.

The test of validity

Example: Is this valid?:

Socrates is a man.

All men are mortal.

∴ Socrates is mortal.

Symbolization:

S

A

∴ M

Truth table:

		S	A	M
	1.	T	T	T
☞	2.	T	T	F
	3.	T	F	T
	4.	T	F	F
	5.	F	T	T
	6.	F	T	F
	7.	F	F	T
	8.	F	F	F

Wait for the predicate calculus.

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Chapter 4: Propositional Logic Proofs

To Discuss Today:

How to do proofs

A bunch of inference rules

Reductio ad absurdum & conditional proof

What Are Inference Rules?

What is a rule of inference?

Implications versus equivalences

Seven Simple Rules

Addition (add):

$$\frac{p}{p \lor q} \qquad \frac{q}{p \lor q}$$

Conjunction (conj):

$$\frac{\begin{array}{l} p \\ q \end{array}}{p \,\&\, q}$$

Commutative Law (comm):

$p \,\&\, q \equiv q \,\&\, p$

$p \lor q \equiv q \lor p$

Double Negation (DN):

$p \equiv {\sim}{\sim}p$

Material Implication (impl):

$p \to q \equiv {\sim}p \lor q$

Material Equivalence (equiv):

$p \leftrightarrow q \equiv (p \to q) \,\&\, (q \to p)$

$p \leftrightarrow q \equiv (p \,\&\, q) \lor ({\sim}p \,\&\, {\sim}q)$

DeMorgan's Law (DeM)

${\sim}(p \,\&\, q) \equiv ({\sim}p \lor {\sim}q)$

${\sim}(p \lor q) \equiv ({\sim}p \,\&\, {\sim}q)$

Using the rules in a proof

Example:

Given: ~A, B.

To prove: ~(B → A).

1.	~A	premise

2.	B	premise
3.	~A & B	1,2 conj
4.	~~(~A & B)	3 DN
5.	~(~~A ∨ ~B)	4 DeM
6.	~(A ∨ ~B)	5 DN
7.	~(~B ∨ A)	6 comm
8.	~(B → A)	7 impl

Reductio ad Absurdum & Conditional Proof

The idea of reductio ad absurdum

p	~p
⋮	⋮
q & ~q	q & ~q
~p	p

The idea of conditional proof

p
⋮
q
———
p → q

Examples: proofs for 3 famous laws of logic

Example 1: Law of Excluded Middle: A ∨ ~A.

→1.	~(A ∨ ~A)	Assumption
2.	~A & ~~A	1 DeM
3.	(A ∨ ~A)	1-2 RAA

Example 2: Law of Non-Contradiction: ~(A & ~A).

→1.	(A & ~A)	a.
2.	~(A & ~A)	1-1 RAA

Example 3: Not really the Law of Identity: (A ↔ A).

→	1. A	a.
	2. $A \rightarrow A$	1-1 CP
	3. $(A \rightarrow A) \& (A \rightarrow A)$	2,2 conj.
	4. $(A \leftrightarrow A)$	3 equiv.

Using assumptions properly

Rules for use of assumptions:

- All assumptions must be discharged
- After assum. is discharged: Do not use steps from inside its scope
- Conclusion must be outside the scope of any assumptions
- Using multiple assumptions: The brackets should not cross

Example: What is wrong with this?

	1. A	premise
→	2. ~A	a.
	3. A & ~A	1,2 conj
	4. A	2-3 RAA
→	5. ~B	a.
	6. A & ~A	1,2 conj
	7. B	5-6 RAA

More Rules of Inference

Disjunctive Syllogism (DS):

$$\frac{p \vee q \quad \sim p}{q} \qquad \frac{p \vee q \quad \sim q}{p}$$

Modus Ponens (MP):

$$\frac{p \rightarrow q \quad p}{q}$$

Simplification (simp):

$$\frac{p \,\&\, q}{p} \qquad \frac{p \,\&\, q}{q}$$

Exportation (exp):

$(p \& q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

Modus Tollens (MT):

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \sim p \end{array}$$

Hypothetical Syllogism (HS):

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline p \rightarrow r \end{array}$$

Constructive Dilemma (CD):

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ p \vee r \\ \hline q \vee s \end{array}$$

Contraposition (contra):

$p \rightarrow q \equiv \sim q \rightarrow \sim p$

Tautology (taut):

$p \equiv p \vee p$

$p \equiv p \& p$

Associative Law (assoc):

$p \& (q \& r) \equiv (p \& q) \& r$

$p \vee (q \vee r) \equiv (p \vee q) \vee r$

Distributive Law (dist):

$p \& (q \vee r) \equiv (p \& q) \vee (p \& r)$

$p \vee (q \& r) \equiv (p \vee q) \& (p \vee r)$

Miscellaneous Stuff

Theorems of propositional logic

- What are they?

Proof strategy

- Memorize all the rules
- Start by writing premises
- RAA and CP are very useful. CP for proving conditionals.
- Look for premises that haven't been used
- Try to get A, ~A, B, ~B, etc.
- ~(A v B) or ~(A & B): use DeM
- (A & B): use Simp.
- (A → B): use MP or Impl.

~(A → B): convert to A and ~B
Work backwards
Combine the last step with something earlier
Look back at previous steps for pairs that can be combined
Always remember the conclusion
Check your work over