1. Introduction

This note presents a simple, analytical method for the design of cassegrain baffle systems that can easily be programmed in a spreadsheet or simple PC based program. There are a number of sources in the ATM and professional literature describing graphical procedures for the layout of cassegrain baffles (e.g. Rutten & van Venrooij 1988 (§19.3), Willey 1963, Prescott 1968). I am aware of only two papers giving quantitative procedures: Young (1967) and Hales (1992). Wilson (1999) appears to have missed Hales' work and instead outlines a graphical procedure that he credits to Prescott. Young describes an iterative procedure that is readily programmable, but that does require a good starting guess for guaranteed convergence. Hales takes a similar basic approach to Young, but by manipulating the equations describing the baffle positions he derives a quartic equation for one of the baffle coordinates, from which the remaining coordinates are solved in a sequence of substitutions. Somewhat surprisingly, Wilson accepts Young's claim that no analytical solution exists for baffle layout. Hales shows this to be false, and in fact the algebraic procedure I outline here should give identical results to Wilson's graphical procedure.

In trying to work through Hales' paper I noticed a couple practical difficulties. First, he attempts to derive an "exact" solution using an exact raytrace for spherical elements – aside from the fact that this is itself an approximation it complicates (unnecessarily I think) the analysis considerably. Second, I found at least one or two undefined parameters in the paper, which makes actually implementing his procedure problematic. Finally, there is a subtle source of vignetting in the way he sets up his baffle layout. The effect would be minor in practice, but it is an error that is not found in Young's paper, for example.

2. Setup

This work takes a similar analytical approach to Hales, but I achieve a major simplification by using a strictly paraxial analysis of the telescope layout. Treating the mirrors as effectively plane eliminates a set of second order terms from the equations, and in the final analysis we end up with only a quadratic equation to solve. Using a paraxial approximation appears to be entirely adequate in practice, especially for amateur projects that are unlikely to have primaries faster than around f/2.5. An exact raytrace of the actual system design can be used to refine the final baffle positions if necessary. The few cases I've looked at would require very minor adjustments.

Note that in general an "optimum" baffle system as defined in section 2.2 below will require baffles on both the secondary and primary. The secondary baffle will always be conical in shape, opening outward from the secondary position. The primary baffle may be conical or cylindrical. I will consider some possible design variations in section 4.2 below.

In this paper I will also take care to define all parameters and use consistent, modern conventions for notation and signs. A companion to this paper is an Excel spreadsheet for general cassegrain design and baffle layout. That is described more fully in section 4.

2.1 Definitions: Notation and sign conventions

In contrast to much of modern physics there are no universally agreed upon conventions for signs or commonly used symbols in geometric optics. I favor the notational conventions adopted by Wilson (1996), mostly because he is careful and thorough in his definitions and also because the twin volumes of "Reflecting Telescope Optics" are likely to remain the standard reference for telescope design for years to come. In particular I follow Wilson in using a "strict Cartesian" sign convention – this is consistent with the practice of most modern lens design software. Also in keeping with modern lens design conventions the optical axis of the system is coincident with the z-axis, with the optical elements parallel to the x-y plane. Since we only need to trace "meridional" rays we define the vertical axis to be the y-axis; the x-axis is unneeded. The vertex of the primary mirror is taken to be the origin.

Table 1 lists the parameters necessary for the analysis. All of these are from the paraxial solution for a cassegrain telescope, and may be either design or derived parameters. The analysis could be carried out in terms of other sets of parameters, but these are convenient and should be available as part of any telescope design solution. In the following table signs are indicated as "+" or "-", where "+" is understood to mean "≥ 0 ".

Parameter	Sign	Definition
y_1	+	Primary semi-diameter
f_1'	-	Primary focal length
b		Primary - focal plane separation. This is normally positive for focal plane located behind primary, but could be either sign.
d	-	Primary - secondary separation
y_2	+	Secondary semi-diameter. This includes the supplement necessary to fully illuminate a field of size y_F. If y_A is the minimum required secondary size, $y_2 = y_A - d\, y_F / f'$ in the paraxial approximation. Note: In Wilson y_2 is the minimum secondary diameter.
m_2	-	Secondary magnification
f'	+	System effective focal length = $m_2 f_1'$.
y_F	+	Linear size of fully illuminated field, at the focal plane. See y_2 above
u_{pr}	+	Angular field semi-diameter (in radians); $u_{pr} = y_F / f'$.

Table 1

In addition to these we also define y_S as the linear semi-diameter at the focal plane of the fully shielded field. In general we need not have $|y_S| = |y_F|$; if some portion of the field must be fully shielded from stray light we will have $y_S < 0$. If $y_S = y_F$ the optimal solution will have only a primary baffle.

2.2 Problem definition

The conceptual idea behind the following analysis is quite simple. According to Hales there are 3 conditions for an "optimum" baffle system: (1) Direct rays of stray light must be eliminated for a specified area parameterized by y_S; (2) A field defined by y_F must be uniformly illuminated; and (3) the baffles must introduce minimal obstruction consistent with (1) and (2). In practice we just need to trace two full field rays plus a single stray light ray through the system to determine the baffle positions.

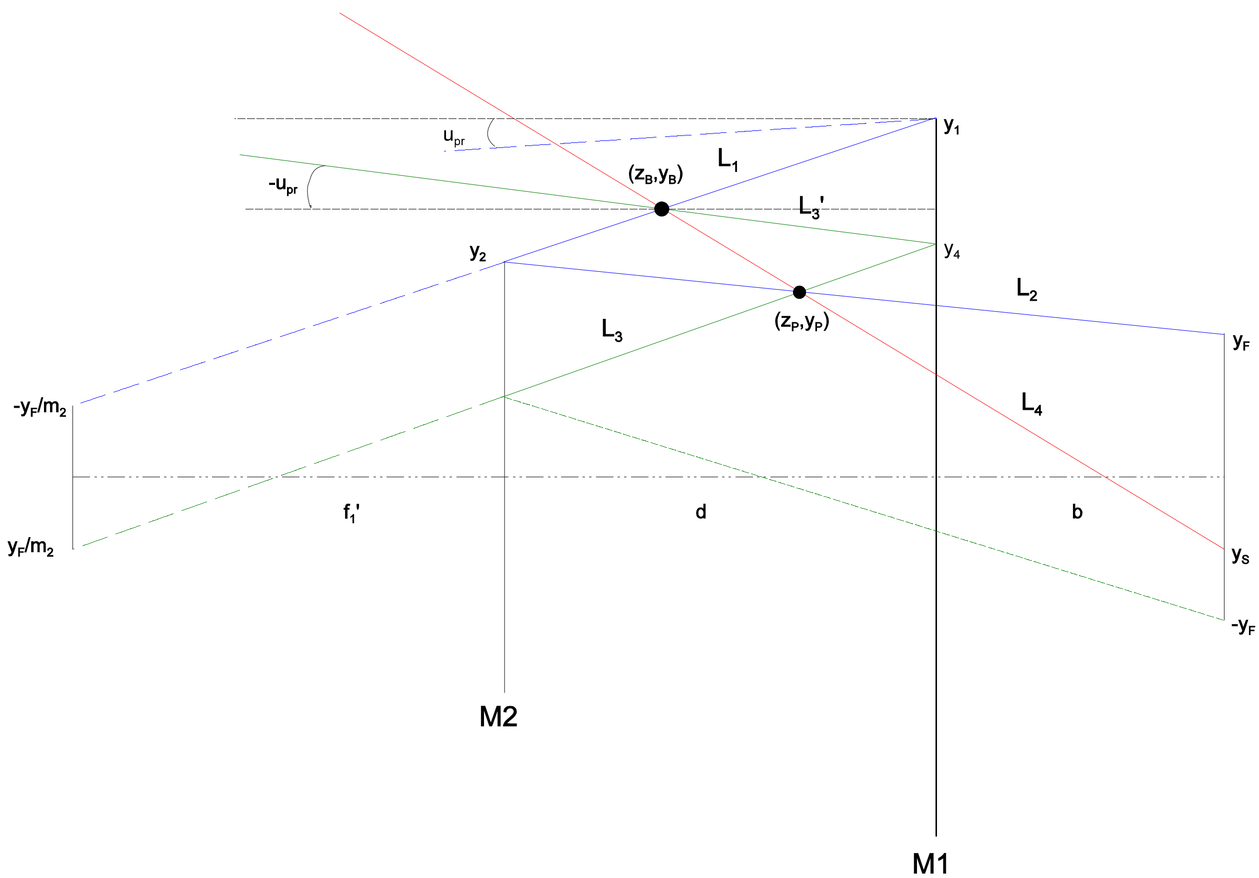

Figure 1

Figure 1 shows in graphical form everything we need for the analysis. First, the "uniformly illuminated" condition implies that all rays must pass through both baffle openings. In particular the marginal full field ray must reach the focal plane, and this defines the minimum bounds on each baffle. This ray is shown as the line segments L_1 and L_2 in Figure 1.

The line segment L_4 represents a stray light ray. The points where it intersects the line segments L_1 and L_2 define the endpoints of the secondary and primary baffle, respectively. It should be obvious from the drawing that for any other position in the focal plane with $|y| \leq |y_S|$ no stray light can reach the focal plane.

Finally, the minimum vignetting condition is met as follows: Consider the full field ray represented by the line segment L_3' that just clears the secondary baffle and reflects off the primary mirror at position $y_{4.}$ That ray must clear the primary baffle on its way to the secondary (represented by L_3), and that in turn implies that all three line segments L_2, L_3, and L_4 must intersect at the position (z_P, y_P), which defines the endpoint of the primary baffle.

3. Solution

Hales' basic insight was to write out linear equations representing the 5 ray segments that need to be traced, and derive by a sequence of substitutions an expression in one variable for one of the baffle coordinates. It turns out that there are 4 independent variables here – the coordinates (z_B, y_B) and (z_P, y_P) representing the endpoints of the secondary and primary baffles respectively – and 4 equations defining the relations between them. The coordinate y_4 where the innermost full field ray intercepts the primary is also unknown, but completely determined by the secondary baffle position.

We follow Hales in writing the ray equations in the form $y = \alpha_i + \beta_i z$ for the line segment L_i. These can be written down easily using Figure 1:

$$L_1\text{: } y = y_1 + \left(\frac{y_2 - y_1}{d}\right) z \tag{1}$$

$$L_2\text{: } y = \left(\frac{y_2 b - y_F d}{b-d}\right) + \left(\frac{y_F - y_2}{b - d}\right) z \tag{2}$$

$$L_3'\text{: } y_4 = y_B + u_{pr} z_B \tag{3}$$

$$L_3\text{: } y = y_4 + \left(\frac{y_F / m_2 - y_4}{f_1'}\right) z = y_4 + (y_F / f' - y_4 / f_1')z \tag{4}$$

The line segment L_4 is slightly more complicated, since it passes through both points (z_B, y_B) and (z_P, y_P). We choose to define its slope and intercept in terms of the primary baffle position:

$$L_4\text{: } y = \left(\frac{y_P b - y_S z_P}{b - z_P}\right) + \left(\frac{y_S - y_P}{b - z_P}\right) z \tag{5}$$

In order to simplify the expressions we substitute the symbols β_1, α_2, and β_2 for the known slopes and intercepts in equations (1) and (2) above. We first solve for (z_B, y_B) as the intersection of the line segments L_1 and L_4. With a bit of rearrangement of terms we get:

$$z_B = \frac{y_1(b - z_P) - (y_P b - y_S z_P)}{y_S - y_P - \beta_1(b - z_P)} \tag{6}$$

$$y_B = \frac{y_1(y_S - y_P) - \beta_1(y_P b - y_S z_P)}{y_S - y_P - \beta_1(b - z_P)} \tag{7}$$

Now substitute equations 6 and 7 into equation 3 to get an expression for y_4:

$$y_4 = \frac{y_1(y_S - y_P) - (\beta_1 + u_{pr})(y_P b - y_S z_P) + u_{pr} y_1(b - z_P)}{y_S - y_P - \beta_1(b - z_P)} \tag{8}$$

Next we solve for (z_P, y_p) as the intersection of the line segments L_2 and L_3:

$$z_P = \frac{\alpha_2 - y_4}{y_F / f' - y_4 / f_1' - \beta_2} \tag{9}$$

$$y_P = \frac{\alpha_2(y_F / f' - y_4 / f_1') - \beta_2 y_4}{y_F / f' - y_4 / f_1' - \beta_2} \tag{10}$$

Now we are getting close. In equation 8 we have an expression for y_4 in terms of the unknowns z_P and y_P, while equations 9 and 10 give expressions for z_P and y_P in terms of y_4. Substituting (9) and (10) into 8 we get an equation of the form

$$y_4 = N(y_4) / D(y_4)$$

with the numerator and denominator both linear functions of y_4. Rearranging this gives a quadratic equation in y_4:

$$a y_4^2 + b y_4 + c = 0$$

where:

$$
\begin{aligned}
a &= -y_S / f_1' + \alpha_2 / f_1' + \beta_2 + \beta_1 b / f_1' - \beta_1 \\
b &= (y_F / f' - \beta_2)(y_S - \beta_1 b) + \alpha_2 (\beta_1 - y_F / f' - y_1 / f_1') \\
&\quad + y_1 y_S / f_1' - \beta_2 y_1 + (\beta_1 + u_{pr})(y_S - b\alpha_2 / f_1' - b\beta_2) \\
&\quad + u_{pr} y_1 (b / f_1' - 1) \\
c &= -y_1 y_S (y_F / f' - \beta_2) + \alpha_2 y_1 y_F / f' \\
&\quad + \alpha_2 (\beta_1 + u_{pr})(b y_F / f' - y_S) \\
&\quad + u_{pr} y_1 (\alpha_2 - b(y_F / f' - \beta_2))
\end{aligned} \tag{11}
$$

The solution for the baffle end positions is now obtained as follows. Solve for y_4 using the parameters in (11) – in every case that I've examined so far the roots are real and the larger root gives the desired solution. Next substitute into (9) and (10) to get z_P and y_p. Then substitute in (6) and (7) to get the secondary baffle position (z_B, y_B).

4. Comments

One obvious question concerns the adequacy of the paraxial approximation used here. The best way to check in any given design is to run an exact raytrace using the lens design program of one's choice. The one such program (OSLO) that I'm familiar with has no built in facilities for baffle design, at least in its freeware version (that I'm aware of!), but it does allow setting up special apertures with holes or obstructions. Figure 2 shows a fairly extreme example with an f/2 primary, f/5 system and primary-focal plane distance equal to the primary semi-diameter. The field size is set to about 4.6 degrees, or 40% of the primary diameter at the focal plane. Baffle positions are indicated by vertical line segments between the mirror positions. Close examination of the drawing does indicate some vignetting of extreme off-axis rays by both baffles – the fully illuminated field is approximately 90% of the target value. More realistic designs should do much better than this.

4.1 The spreadsheet

As a companion to this paper I have prepared an Excel spreadsheet that is intended as a complete design tool for cassegrain family telescopes. Besides baffle layout it computes the paraxial layout using either of two possible sets of design parameters, and also computes conic constants and Seidel aberration coefficients for classical, RC, and DK variants. The basic design spreadsheet should work for all two mirror, axially symmetric systems including Gregorians and more exotic cassegrain variants such as Schwarzschild and Couder aplanats. I have made no attempt to verify the baffle solutions for "exotic" systems, however it should work for systems in which the focal plane lies in front of the primary and with modifications it can be used for systems with a Nasmyth focus. The procedure given in this paper will **not** work for a Gregorian.

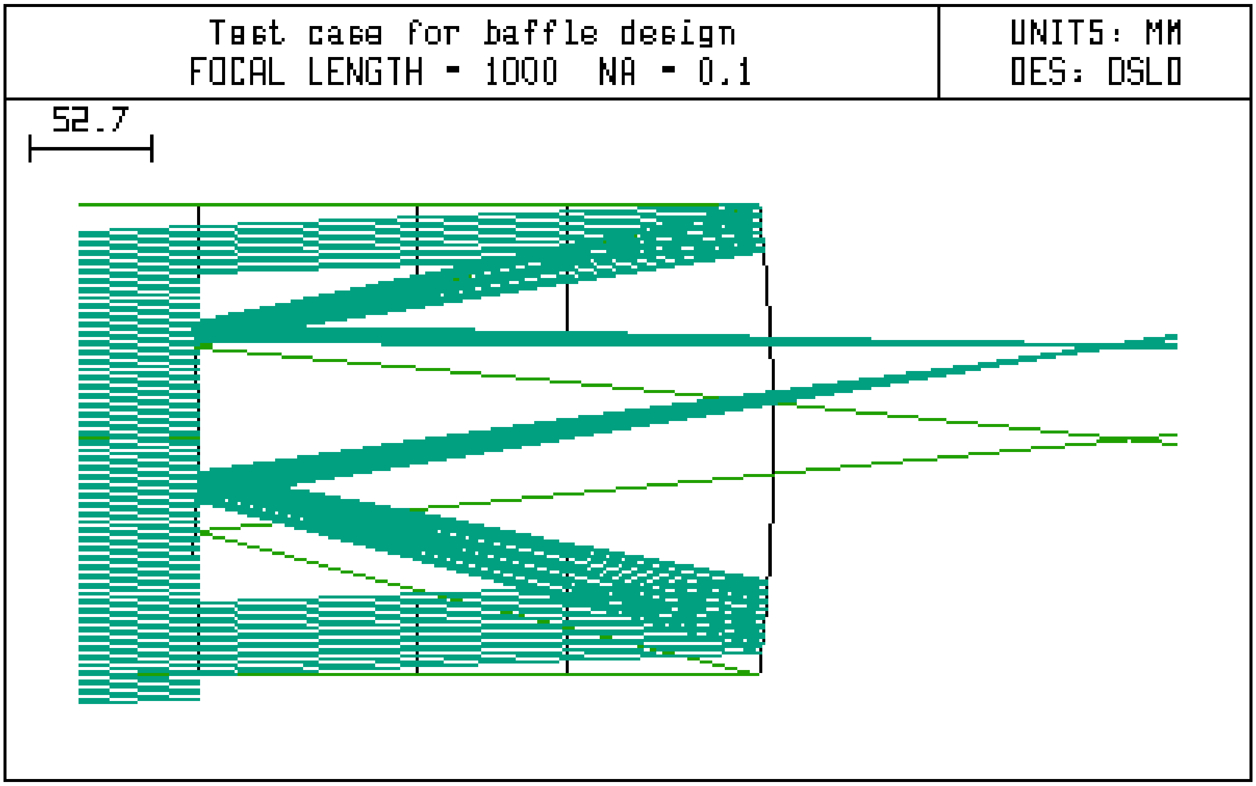

Figure 2

4.2 Possible design variations

For conventional cassegrains the spreadsheet shows the numerical solution for the baffle end points and provides a scale drawing of the layout. The primary baffle is drawn as a segment of a cone, which should be understood as the maximum size of the baffle to meet the no vignetting condition. In Figure 1 it can be seen that the triangular area in front of the primary between the line segments L_2 and L_3 is shadowed by the secondary and its baffle, and the primary baffle tube can occupy any part of this space. Usually, but not always, a cylindrical baffle tube will work. One could also use a sequence of nested cylinders with setbacks. Manufacturing convenience and perhaps consideration of grazing incidence light will dictate the final form of the primary baffle.

The secondary baffle is more troublesome, especially for amateur telescopes where minimizing the secondary obstruction is usually a major objective. A couple reasonable solutions are offered in the ATM literature and often used in commercial cassegrains. One possibility is to do away with the secondary baffle altogether – this case can be calculated in my spreadsheet by setting the shielded field size equal to the illuminated field size. In general the resulting primary baffle will allow some sky flooding at all field positions, but in most situations this may be perfectly acceptable. Another possibility is to use a cylindrical baffle tube with a secondary slightly larger than needed for the desired uniformly illuminated field. This solution may be the most reasonable compromise in a telescope that's going to be used for general visual use plus some ccd imaging, or one that's primarily intended for planetary viewing or other narrow angle targets. In either case it's desirable to have some finite size field that's fully shielded and uniformly illuminated, but that field may be small compared to the total accessible field size. The spreadsheet can be useful for this case as well: just decide what size field must be fully illuminated and shielded and calculate the baffle positions. Then make the secondary size equal to the baffle diameter and the secondary baffle length equal to z_B-d..

As a final comment, I have not considered the case where the system's entrance pupil is at a position other than the primary mirror. In amateur telescopes this is mostly relevant to catadioptric systems, where the corrector element usually serves as the aperture stop. The paraxial layout of an all reflective system might or might not be nearly correct for a catadioptric telescope, depending on the optical power of the corrector. If the corrector has no net power the spreadsheet solution **may** be at least a useful starting point for a more detailed analysis.

References

Hales, W.L. 1992, **Applied Optics**, **31** (no. 25), p. 5341.

Prescott, R. 1968, **Applied Optics, 7** (3), 479.

Rutten, H. and van Venrooij, M. 1988, **"Telescope Optics – Evaluation and Design"**, Willmann-Bell, Richmond, VA.

Willey, R.R. 1963, **Sky and Telescope**, April 1963, p. 232.

Wilson, R.N. 1996, **"Reflecting Telescope Optics I"**, Springer-Verlag, Berlin.

Wilson, R.N. 1999, **"Reflecting Telescope Optics II"**, Springer-Verlag, Berlin.

Young, A.T. 1967, **Applied Optics, 6** (6), 1063.