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Infinite products with strongly $B$-multiplicative exponents. (English summary)


In this paper the authors evaluate a number of interesting infinite products of the form
\[
\prod_{n \geq 0} a(n)^{u(n)},
\]
where $a(n)$ is a rational function and $u(n)$ either is strongly $B$-multiplicative (that is, $u(Bn + k) = u(n)u(k)$) or satisfies a slightly weaker hypothesis. A typical example is the authors’ Example 6:

\[
\prod_{n \geq 0} \left( \frac{2n + 1}{2n + 2} \right)^{(1/2)^{s_{2}(n)}} = \frac{1}{4},
\]

where $s_{k}(n)$ denotes the sum of the digits in the base-$k$ representation of $n$. Unfortunately, there are a number of typographical errors. For example, the subscript on the $\prod$ symbol in Corollaries 2 and 4 should read $k \not\equiv 0 \pmod{q}$.

Reviewed by Jeffrey O. Shallit

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