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MR2167772 (2006d:11156) 11Y60
Sondow, Jonathan
A faster product for π and a new integral for $\ln\frac{\pi}{2}$.
Amer. Math. Monthly **112** (2005), *no. 8,* 729–734.

In ["An infinite product for e^γ via hypergeometric formulas for Euler's constant, γ", preprint, arxiv.org/abs/math/0306008], the author proved the infinite product identity

$$e^\gamma = \left(\frac{2}{1}\right)^{1/2}\left(\frac{2^2}{1\cdot 3}\right)^{1/3}\left(\frac{2^3\cdot 4}{1\cdot 3^3}\right)^{1/4}\left(\frac{2^4\cdot 4^4}{1\cdot 3^6\cdot 5}\right)^{1/5}\cdots,$$

where γ is Euler's constant and the expression of the n-th term under the $(n+1)$-st root is

$$P_n = \prod_{k=0}^{n}(k+1)^{(-1)^{k+1}\binom{n}{k}}.$$

In this note, he shows the identity

$$\frac{\pi}{2} = \left(\frac{2}{1}\right)^{1/2}\left(\frac{2^2}{1\cdot 3}\right)^{1/4}\left(\frac{2^3\cdot 4}{1\cdot 3^3}\right)^{1/8}\left(\frac{2^4\cdot 4^4}{1\cdot 3^6\cdot 5}\right)^{1/16}\cdots,$$

for which he provides three proofs (using respectively the alternate zeta function, Wallis integrals and certain integrals reminiscent of those of Beukers).

He also mentions the existence of a third identity based on P_n and due to J. Guillera:

$$e = \left(\frac{2}{1}\right)^{1/1}\left(\frac{2^2}{1\cdot 3}\right)^{1/2}\left(\frac{2^3\cdot 4}{1\cdot 3^3}\right)^{1/3}\left(\frac{2^4\cdot 4^4}{1\cdot 3^6\cdot 5}\right)^{1/4}\cdots.$$

REVISED (April, 2006)
Current version of review. Go to earlier version.

Reviewed by *Tanguy Rivoal*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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