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Double integrals for Euler's constant and $\ln \frac{4}{\pi}$ and an analog of Hadjicostas's formula.

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The author provides ingenious and intriguing evaluations of the integrals

$$\int_0^1 \int_0^1 \frac{1-x}{1 \pm xy} (\log xy)^s dx dy.$$

For instance, the case $s = -1$ and the choice of minus sign yields the Euler constant γ .

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Current version of review. [Go to earlier version.](#)

Reviewed by *A. J. van der Poorten*

References

1. M. Abramowitz and I. Stegun, eds., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover, New York, 1965. [MR1225604 \(94b:00012\)](#)
2. F. Beukers, A note on the irrationality of $\zeta(2)$ and $\zeta(3)$, *Bull. London Math. Soc.* **11** (1979) 268–272. [MR0554391 \(81j:10045\)](#)
3. G. Boros and V. Moll, *Irresistible Integrals: Symbolics, Analysis, and Experiments in the Evaluation of Integrals*, Cambridge University Press, Cambridge, 2004. [MR2070237 \(2005b:00001\)](#)
4. J. Borwein and P. Borwein, *Pi and the AGM*, John Wiley & Sons, New York, 1987. [MR0877728 \(89a:11134\)](#)
5. R. Chapman, A proof of Hadjicostas's conjecture (2004, preprint); available at <http://arXiv.org/abs/math/0405478>.
6. P. Davis, Leonhard Euler's integral: A historical profile of the gamma function, this Monthly **66** (1959) 849–869. [MR0106810 \(21 #5540\)](#)
7. W. Dunham, *Euler: The Master of Us All*, Dolciani Mathematical Expositions 22, Mathematical Association of America, Washington, D.C., 1999. [MR1669154 \(2000b:01007\)](#)
8. S. Finch, *Mathematical Constants*, Cambridge University Press, Cambridge, 2003. [MR2003519 \(2004i:00001\)](#)
9. P. Hadjicostas, A conjecture-generalization of Sondow's formula (2004, preprint); available at <http://arXiv.org/abs/math/0405423>.
10. P. Hadjicostas, personal communication, 30 May 2004.
11. J. Havil, *Gamma: Exploring Euler's Constant*, Princeton University Press, Princeton, 2003. [MR1968276 \(2004k:11195\)](#)
12. D. Huylebrouck, Similarities in irrationality proofs for π , $\ln 2$, $\zeta(2)$ and $\zeta(3)$, this Monthly **108** (2001) 222–231. [MR1834702 \(2002b:11095\)](#)
13. J. Sondow, Analytic continuation of Riemann's zeta function and values at negative integers

- via Euler's transformation of series, *Proc. Amer. Math. Soc.* **120** (1994) 421–424. [MR1172954 \(94d:11066\)](#)
14. J. Sondow, An antisymmetric formula for Euler's constant, *Math. Mag.* **71** (1998) 219–220.
 15. J. Sondow, Zeros of the alternating zeta function on the line $\Re(s) = 1$, *this Monthly* **110** (2003) 435–437. [MR2040887](#)
 16. J. Sondow, Criteria for irrationality of Euler's constant, *Proc. Amer. Math. Soc.* **131** (2003) 3335–3344. [MR1990621 \(2004b:11102\)](#)
 17. J. Sondow, A faster product for π and a new integral for $\ln(\pi/2)$, *this Monthly* (to appear); available at <http://arXiv.org/abs/math/0401406>.
 18. J. Sondow, An infinite product for e^γ via hypergeometric formulas for Euler's constant, γ (2003, preprint); available at <http://arXiv.org/abs/math/0306008>.
 19. J. J. Sylvester, *Collected Papers*, vol. 2, Chelsea, New York, 1973.
 20. A. van der Poorten, A proof that Euler missed...Apéry's proof of the irrationality of $\zeta(3)$, *Math. Intelligencer* **1** (1979) 195–203. [MR0547748 \(80i:10054\)](#)

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