MR1172954 (94d:11066) 11M06

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Analytic continuation of Riemann's zeta function and values at negative integers via Euler's transformation of series.

Proc. Amer. Math. Soc. **120** (1994), *no. 2,* 421–424.

The convergent alternating series $\sum_1^\infty (-1)^{n-1} a_n$ of complex numbers is equal to $\sum_0^\infty 2^{-j-1}\Delta^j a_1$, where $\Delta^j a_n = \Delta^{j-1}a_n - \Delta^{j-1}a_{n+1}$ if $j \geq 1$ and $\Delta^0 a_n = a_n$. This is Euler's transformation of series. In this note the author chooses $a_n = n^{-s}$ and, using a certain formula for Bernoulli numbers, obtains the familiar formula for $\zeta(s)$ at negative integers. The method is justified by first showing that $\sum_0^\infty 2^{-j-1}\Delta^j 1^{-s}$ gives the analytic continuation of $(1-2^{1-s})\zeta(s)$ to the entire complex plane.

Reviewed by *Robert J. Anderson*