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Analytic continuation of Riemann’s zeta function and values at negative integers via Euler’s transformation of series.


The convergent alternating series $\sum_{1}^{\infty} (-1)^{n-1}a_n$ of complex numbers is equal to $\sum_{0}^{\infty} 2^{-j-1} \Delta^j a_1$, where $\Delta^j a_n = \Delta^{j-1} a_n - \Delta^{j-1} a_{n+1}$ if $j \geq 1$ and $\Delta^0 a_n = a_n$. This is Euler’s transformation of series. In this note the author chooses $a_n = n^{-s}$ and, using a certain formula for Bernoulli numbers, obtains the familiar formula for $\zeta(s)$ at negative integers. The method is justified by first showing that $\sum_{0}^{\infty} 2^{-j-1} \Delta^j 1^{-s}$ gives the analytic continuation of $(1 - 2^{1-s})\zeta(s)$ to the entire complex plane.

Reviewed by Robert J. Anderson

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