On Dark Energy, Inflation, and the Cosmological Constant

Richard P. Dolan

Abstract

In the inflaton spacetime model, spacetime is an expanding set of self-reproducing quantum states called points. The expansion is very rapid during an initial inflationary period. It becomes much slower, although still accelerating, after a phase transition. Dark energy, the inflaton, and the cosmological constant are just different aspects of the expansion of spacetime. I describe the mechanism of expansion and show what one would have to do to compute the cosmological constant based on this model.

Cosmic inflation was invented in 1981 to solve a number of problems that occur in the standard big bang theory of the origin of the universe. While the theory is spectacularly successful at solving these problems and has become an important part of the standard cosmology, it has proved extremely difficult to find a physical realization of the theory that fits the data exactly. All inflationary models proposed so far have many problems of their own. Yet the data, especially measurements of the cosmic microwave background, or CMB, firmly supports the inflationary scenario.

The inflaton spacetime model (*Physics Essays*, **19**, 370 and here) is a discrete quantum spacetime model underlying the standard models of particle physics and cosmology. It is an inflationary model, but the inflationary scenario it describes is very different from the models proposed so far and does not have their problems.

Here is an "executive summary."

What is Spacetime?

Spacetime is the set of all states of existence, that is, the set of all states in which existence is possible. The states are called points. Points are quantum objects described by quantum numbers. The principal quantum numbers are called position and spin. Points are all that exist. There is no continuous background space in which positions have meaning. Position, like spin, is an *intrinsic* quantum number. Positions are completely random, within limits. Points are discrete objects, so space is discrete. Between points is nothing, the void. It is postulated that there are at least two points.

The Big Bang

Points are self-reproducing. Every set of points gives rise to a new point. Points are identical, indistinguishable. N points produce 2^N-1 points. N of these are images of the original points and the others are new. Each advance from N points to 2^N-1 defines a time tick. Thus time, like space, is discrete. Time is an intrinsic quantum number of a point. Between time ticks, there is no need for an intermediate time. Thus there is really no time, just as there is no space. There are only intrinsic quantum numbers of points.

The number of points quickly becomes astronomical as the time quantum number increases. This is the Big Bang. On each time tick, the position of a given point is random, but since spacetime must look the same from every point, positions must lie on a three-sphere (in 3D).

Spin, Statistics, and Gravity

All points are superpositions, or mixed states: fermions on some observations (time ticks) and bosons on others. Because you can’t tell whether spacetime is one field of mixed-state points or two fields of points, one fermionic and one bosonic (points are indistinguishable), spacetime can be viewed as two fields, which are coupled because of mixing.

Two fermionic points cannot occupy the same position. Bosonic points try to occupy the same position. As the bosonic points seek the same state, they drag the fermionic points with them because the two fields are coupled as a result of mixing. This quantum mechanical tendency is gravity. It is latent in spacetime. Particles break the symmetry and make it observable. When the fermionic points are as close together as they can be on average, spacetime is stable, with both types of points uniformly distributed.

Inflation

Initially, points are few and far between. The 3-sphere is very large. With each time tick, there are more points, and they are pulled together by gravity, so the 3-sphere both shrinks and grows. The mean free path of a fermionic point shrinks. It can’t be zero, so when it is very small the fermionic points feel degeneracy pressure. When the degeneracy pressure overcomes gravity, spacetime can’t shrink any more. A crash-like phase transition occurs and spacetime is left in a state of coherent oscillations. Each fermionic point is now trapped in a small cell within which its position can vary. The size of the cell is oscillating.

At the phase transition, the probability that a new point is fermionic is sharply reduced because the exclusion principle suppresses fermionic point creation if there is no room.

The expansion continues because quantum position fluctuations always make room for a few more points, but it slows drastically. Most new points are then bosonic.

Consider each fermionic point a particle in a small cell or box. The final size of the box is Planck-scale. The box walls are invisible and the points are identical. The expansion gradually damps out the wall oscillations, leaving some points at their ground state energy while others are in an excited quantum energy state. Particles are points in an excited state. Thus inflation ends in particle production.

The result of this inflation scenario is the same as if the fermionic points were in Planck-scale boxes from the beginning and simply increased in number. All results are the same as in conventional models, such as quantum density fluctuations that seed large-scale structure. But…the conceptual problems of conventional models are absent. The crash-like end of inflation naturally confines density fluctuations to a low level, such as 10^{-5} (amplitude problem). It may also suppress fluctuations at low frequencies/long wavelengths (quadrupole anomaly). The Planck length is established only at the end of inflation. Thus no present scale started out smaller than the Planck length (trans-Planckian problem). There is a kind of initial singularity, but it has no fixed location (singularity problem).

Cosmological Constant

The vacuum energy is the ground-state position fluctuations of the points. These are independent random variables and average to near zero over any significant volume. Thus, vacuum energy does not contribute to the cosmological constant. The cosmological constant, or dark energy, causing the accelerated expansion observed today is the continuing post-inflation expansion in the number of fermionic points, which is relatively slow compared to the inflationary period. Spacetime is very flat after inflation, and since the expansion continues, it is continually driven to flatness. $\Omega = \Omega_m + \Omega_\Lambda = 1$. No fine tuning is necessary. Ω_Λ tracks Ω_m, increasing as Ω_m decreases.

Spacetime is flat regardless of the amount of matter in it. Thus, matter only matters to matter.

Some Cosmological Constant Calculations

At inflation's end, the existing fermionic points are as close together as they can be, on average, so you might think that no more fermionic points could be created (the exclusion principle). However, the quantum fluctuations in the positions of individual points guarantee that there is always room for a few more points, so spacetime continues to expand. The expansion is now much slower than the inflationary expansion, but it is still ultimately an accelerating expansion because, ignoring the presence of matter for the moment, the density of fermionic points and the amplitude to create new fermionic points remain constant. Thus, the dark energy after inflation represents a true cosmological constant, that is, the dark energy density remains constant as the universe expands. The number of fermionic points grows by a fixed (very small) percentage at each time tick.

This is like compound interest, which as everyone knows, leads to an accelerating expansion of one's bank account.

Let’s call the factor by which the volume of the universe grows on each time tick I, for interest. Then if V is the volume of the universe and t is time in seconds,

$V = C(1 + I)^{bt}$,

where C is a proportionality constant and $b = 10^{43}$ is the number of time ticks in one second. Because I is very small, then to a very good approximation, we can write

$V = C(1 + 10^{43}I)^{t}$.

The cosmological constant has been measured, so we can use its known value to find out what I actually is. Ignoring matter and radiation and just looking at the dark energy expansion, the universe fits the de Sitter model, which is a flat Friedman-LeMaitre-Robertson-Walker spacetime that expands according to:

$R = A\exp[(\Lambda c^2/3)^{1/2}t]$,

Where R is the scale factor of the universe, A is a proportionality constant, c is the speed of light, and Λ is the cosmological constant. The volume of the universe is proportional to R^3, so we have

$V = A^3\exp[(3\Lambda c^2)^{1/2}t]$.

We identify C with A^3. For very small x, $e^x \approx (1 + x)$, so we have:

$(1 + 10^{43}I)^t = [1 + (3\Lambda c^2)^{1/2}]^t$, and therefore

$(3\Lambda c^2)^{1/2} = 10^{43}I$.

The measured value of the cosmological constant Λc^2 is approximately $(1/3)10^{-34}\,s^{-2}$.

Thus $I = 10^{-60}$, so the volume of the universe increases by a factor of 10^{-60} on each time tick, or a factor of 10^{-17} per second.

Predicting the Cosmological Constant

Now we would like to go the other way and find a way to calculate I based on fundamental physics so that we can arrive at a prediction for Λ. The calculation is straightforward in principle, but very difficult in practice.

In the inflaton spacetime model, the fermionic points form a lattice in which they are pulled together by gravity and held apart by degeneracy pressure. Each point can be imagined as occupying a roughly spherical cell with walls of infinite potential energy consisting of nearby points. The radius of a point cell is roughly l_{Pl}, where l_{Pl} is the Planck length. The positions of the points exhibit quantum fluctuations within their cells on successive time ticks. The average distance between points is l_{Pl}, so the cells overlap and the wall locations also exhibit random fluctuations. The wave function of a point within its cell and the wave function of a wall of the cell are the same because the walls are other points. To compute the cosmological constant, one needs to know this wave function, and therein lies the extreme difficulty.

Although the points are as close together as they can be on average, their positions fluctuate, so there is some probability that on any given time tick, a point-free region will open up that is big enough for another fermionic point. If this opening sticks around for another time tick, a new fermionic point will be created and the volume of the universe will expand by roughly the volume of a sphere of radius $l_{Pl}/2$ (not l_{Pl}, since the cells overlap).

Let's say that on any given time tick there are N regions in the universe that have a reasonable chance of opening up on the next time tick. Let's call p the probability that any one of these actually opens up, and call p_m the probability that exactly m of them open up, where m = 0, 1, …, N. We have a binomial distribution, for which the expected value of m is

$<m> = Np$.

But I, our "interest rate" for the expansion is simply $<m>/N$, so

$I = p = 10^{-60}$.

Notice that I is independent of N, so the universe expands by the same factor on each time tick. The dark energy is a true cosmological constant.

So we now know that I = p. What does this tell us? It says that if we can find the value of p, the probability that at a given location a point-free region of space appears that is big enough for a new fermionic point, we can predict the value of Λ, the cosmological constant.

What does it take for such a point-free region to appear? First of all, point cells are roughly spherical and are packed as densely as possible. The kissing number is the number of spheres that can touch a sphere of the same size; in three dimensions it is 12. Therefore, the positions of twelve points must fluctuate out of the region in question to make room for a new point. Since point positions are statistically independent, the probability that twelve points get out of the way is the twelfth power of the probability that a single point gets out of the way. What's more, the twelve points must stay out of the way for two time ticks. This squares the probability, so p is the 24th power of the probability p_1 that a single point gets out of the way. This means that $p_1 = 10^{-2.5}$, or about 1/316.

To predict this number we need to find the wave function of a point—any point, since they're all the same. This is a formidable problem. The wave function of a particle in an infinite spherical well is known, but in this case the well walls are moving, and have the same position wave function as the point inside the well! We might approximate a solution by considering that any point's get-out-of-the-way movement consists of three orthogonal movements, thereby reducing the three-dimensional problem to one dimension, but we still have an infinite well with moving walls that are other points and therefore have the same wave function as the enclosed point. I haven't solved this problem, but it is obviously possible in principle.

Even without knowing the exact three-dimensional or one-dimensional wave function of a point, we can still make a rough estimate of p_1. Let's consider the required deviation of a point from its average position to be composed of three independent, equal, and orthogonal random movements. Each point moves within a roughly spherical cell of radius the Planck length. The maximum distance any point would have to move along any axis is roughly half this radius. We can assume that the wave function of appoint is peaked around the average position. We know this is true for the ground state wave function in a one-dimensional infinite square well with fixed walls, and we can assume that the wave function is even more sharply peaked for moving walls. Let's just guess that the point position is within $l_{Pl}/2$ of the average position twice as frequently as it is beyond that distance, say 70% within and 30% beyond, to be specific. Then the point is far enough out of the way along any one axis at least 15% of the time. It is far enough out of the way in three dimensions with probability $p_1 = 0.15^3 = 10^{-2.5}$. Then $p = p_1^{24}$, or $p = (5)10^{-60}$. This happens to be close to the right answer, but we got it without cheating, simply by making some very reasonable assumptions. This shows that the inflaton spacetime model does make it possible to predict the value of the cosmological constant.