Frequency Domain Steerable Pyramid Filter Design

Third derivative, k = 4 filter   Ken Castleman 2/17/98   StPyr_04.mcd

$N := 16 \qquad a := \text{floor}\left(\frac{N}{2}\right) \qquad a = 8 \qquad i := 0..\,N \qquad k := 0..\,N \qquad j := \sqrt{-1}$

Define smooth-edged lowpass and highpass transfer functions (raised cosine).

$$LP(x1,x2,x) := \text{if}\left[x < x1, 1, \text{if}\left[x > x2, 0, \sqrt{0.5\cdot\left[1+\cos\left[\pi\cdot\left(\frac{x-x1}{x2-x1}\right)\right]\right]}\right]\right]$$

$$HP(x1,x2,x) := \text{if}\left[x < x1, 0, \text{if}\left[x > x2, 1, \sqrt{0.5\cdot\left[1-\cos\left[\pi\cdot\left(\frac{x-x1}{x2-x1}\right)\right]\right]}\right]\right]$$

$$\theta_{i,k} := \text{angle}(i + 0.001 - a, k - a)$$

$$\rho_{i,k} := \sqrt{(i-a)^2 + (k-a)^2} \qquad c := \sqrt{\frac{4}{5}}$$

$$f1 := 0\cdot a \qquad f2 := \frac{5}{8}\cdot a \qquad f3 := \frac{6}{8}\cdot a \qquad f4 := 1.2\cdot a$$

Define the transfer functions of the one highpass, two lowpass and three bandpass filters.
The constants f1 and f2 control the steepness of the cutoffs, a is the folding frequency, and c is a required fudge factor.
The LP(f3,f4,f) filter is not required, but it makes the kernels smaller. Use f3 = 7a/8 for perfect reconstruction.

$$B1_{i,k} := c\cdot LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(0, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k}\right)^3$$

$$B2_{i,k} := c\cdot LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(0, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k} - \frac{\pi}{4}\right)^3 \qquad L0_{i,k} := LP\left(f2, a, \rho_{i,k}\right)$$

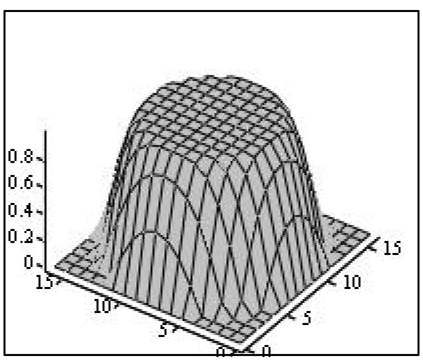
$$B3_{i,k} := c\cdot LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(0, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k} - \frac{\pi}{2}\right)^3 \qquad H0_{i,k} := HP\left(f2, a, \rho_{i,k}\right)$$

$$B4_{i,k} := c\cdot LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(0, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k} - \frac{3\cdot\pi}{4}\right)^3 \qquad L1_{i,k} := LP\left(f1, \frac{a}{2}, \rho_{i,k}\right)$$

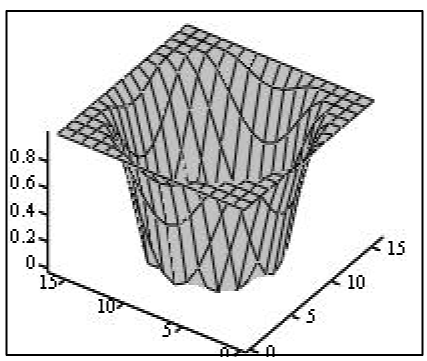
Compute functions to verify that the constraints are satisfied.

$$M1_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2 \quad M1_{N,N} := 0 \quad M2_{i,k} := \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2 + \left(B3_{i,k}\right)^2 + \left(B4_{i,k}\right)^2 \quad M3_{i,k} := M2_{i,k} + \left(L1_{i,k}\right)^2$$

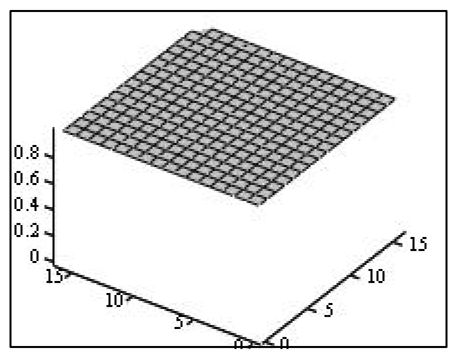
Plot $L_0(u,v)$, $H_0(u,v)$ and the sum of their squared magnitudes.



L0

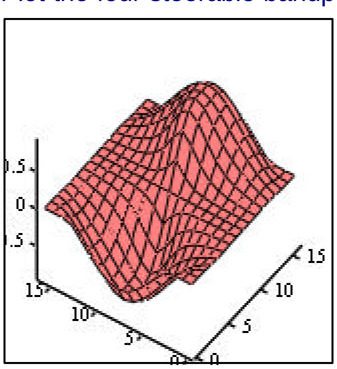


H0

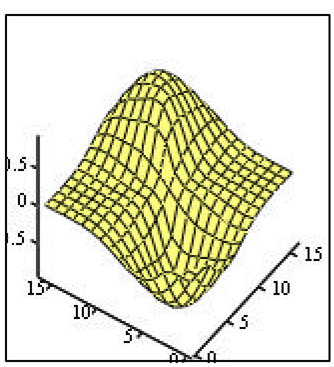


M1

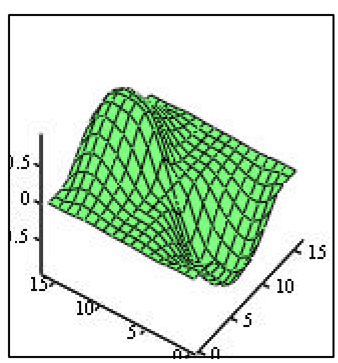
Plot the four steerable bandpass filter transfer functionss.



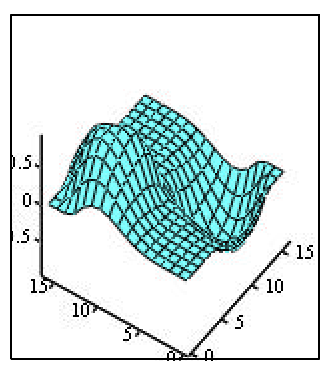
B1



B2

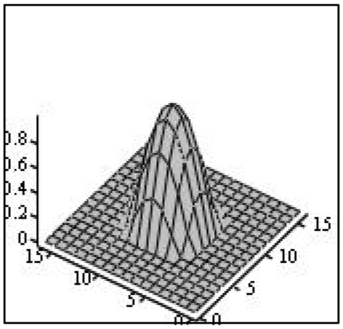


B3

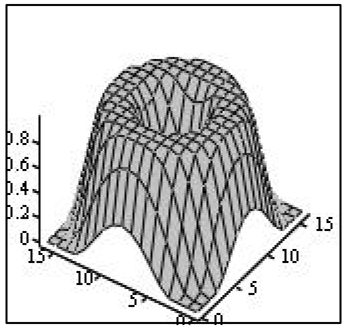


B4

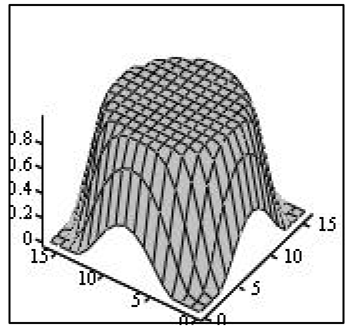
Plot $L_1(u,v)$, the sum the squared magnitude of the four bandpass filters and the grand sum.



L1



M2

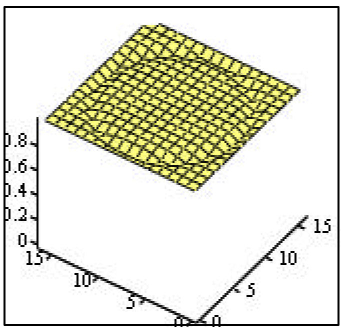


M3

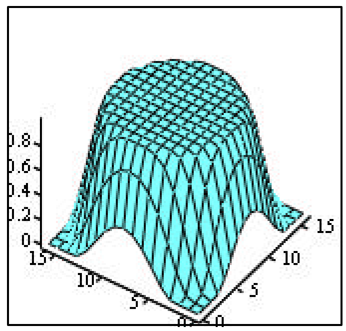
$$M4_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2 \cdot \left[\left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2 + \left(B3_{i,k}\right)^2 + \left(B4_{i,k}\right)^2\right]$$

M4 = 1 is Simoncelli's constraint.

$$M5_{i,k} := \left[\left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2 + \left(B3_{i,k}\right)^2 + \left(B4_{i,k}\right)^2\right] \qquad M4_{N,N} := 0 \quad M5_{N,N} := 0$$



M4



M5

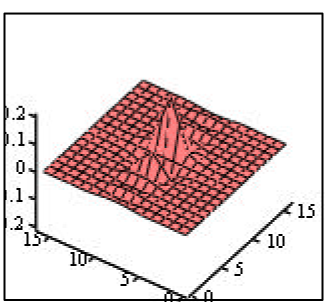
Compute the convolution kernels using the centered DFT.

$$W_{i,k} := \frac{1}{N+1} \cdot \exp\left[-j \cdot 2 \cdot \pi \cdot (i-a) \cdot \frac{k-a}{N+1}\right]$$

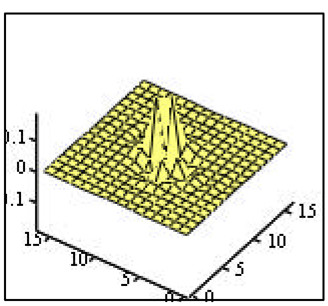
$b1 := W \cdot (j \cdot B1) \cdot W \qquad b2 := W \cdot (j \cdot B2) \cdot W \qquad b3 := W \cdot (j \cdot B3) \cdot W \qquad b4 := W \cdot (j \cdot B4) \cdot W$

$h0 := W \cdot H0 \cdot W \qquad l0 := W \cdot L0 \cdot W \qquad l1 := W \cdot L1 \cdot W$

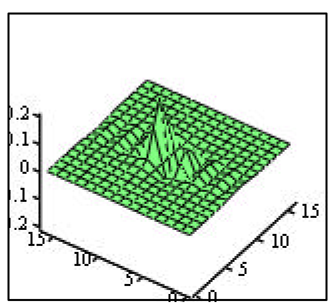
Plot the four bandpass filter impulse responses.



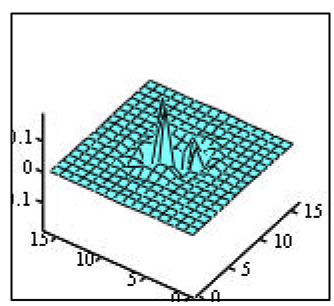
b1



b2

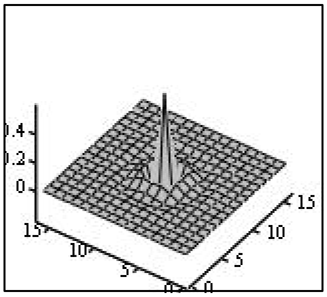


b3

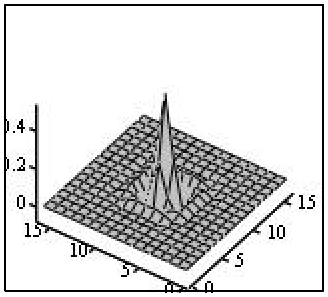


b4

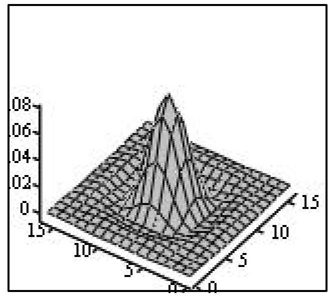
Plot the highpass and the two lowpass filter impulse responses.



h0



l0



l1

$$d := \begin{pmatrix} \frac{N+1}{a} & \frac{N+1}{a} \end{pmatrix}$$

$$d = \begin{pmatrix} \frac{17}{8} & \frac{17}{8} \end{pmatrix}$$

Round off to four digits and scale the values for writing out the kernels.

$b := 1$

$$K4\_L0_{i,k} := \frac{\text{floor}\left(10000\cdot \text{Re}\left(l0_{i,k}\right) + 0.5\right)}{b} \quad K4\_L1_{i,k} := \frac{\text{floor}\left(10000\cdot \text{Re}\left(l1_{i,k}\right) + 0.5\right)}{b} \quad K4\_H0_{i,k} := \frac{\text{floor}\left(10000\cdot \text{Re}\left(h0_{i,k}\right) + 0.5\right)}{b}$$

$$K4\_B1_{i,k} := \frac{\text{floor}\left(10000\cdot \text{Re}\left(b1_{i,k}\right) + 0.5\right)}{b} \quad K4\_B2_{i,k} := \frac{\text{floor}\left(10000\cdot \text{Re}\left(b2_{i,k}\right) + 0.5\right)}{b} \quad K4\_B3_{i,k} := \frac{\text{floor}\left(10000\cdot \text{Re}\left(b3_{i,k}\right) + 0.5\right)}{b}$$

$$K4\_B4_{i,k} := \frac{\text{floor}\left(10000\cdot \text{Re}\left(b4_{i,k}\right) + 0.5\right)}{b}$$

Verify that the constraints are met

$$\sum_i \sum_k K4\_L0_{i,k} = 9996 \quad \sum_i \sum_k K4\_L1_{i,k} = 10005 \quad \sum_i \sum_k K4\_H0_{i,k} = 0$$

$$\sum_i \sum_k K4\_B1_{i,k} = 0 \quad \sum_i \sum_k K4\_B2_{i,k} = 0 \quad \sum_i \sum_k K4\_B3_{i,k} = 0 \quad \sum_i \sum_k K4\_B4_{i,k} = 0$$

Write kernel files for input to the WiT 2-D convolution operator (Use b = 10,000 for unscaled kernels).

WRITEPRN("K4_L0.arr") := d APPENDPRN("K4_L0.arr") := K4_L0

WRITEPRN("K4_L1.arr") := d APPENDPRN("K4_L1.arr") := K4_L1

WRITEPRN("K4_H0.arr") := d APPENDPRN("K4_H0.arr") := K4_H0

WRITEPRN("K4_B1.arr") := d APPENDPRN("K4_B1.arr") := K4_B1

WRITEPRN("K4_B2.arr") := d APPENDPRN("K4_B2.arr") := K4_B2

WRITEPRN("K4_B3.arr") := d APPENDPRN("K4_B3.arr") := K4_B3

WRITEPRN("K4_B4.arr") := d APPENDPRN("K4_B4.arr") := K4_B4

1. E. P. Simoncelli, and W. T. Freeman, "The Steerable Pyramid: A Flexible Architecture for Multi-Scale Derivative Computation," *Proc. ICIP-95*: 444-447, 1995.
2. P. J. Burt, and E. H. Adelson, "The Laplacian Pyramid as a Compact Image Code," *IEEE Trans*. **C-31**:532-540, 1983.
3. E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger, "Shiftable Multiscale Transforms," *IEEE Trans*. **IT-38**(2):587-607, 1992.

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