Frequency Domain Steerable Pyramid Filter Design

Second derivative, k = 3 filter   Ken Castleman 2/17/98   StPyr_03.mcd

$N := 16 \qquad a := \text{floor}\left(\frac{N}{2}\right) \qquad i := 0..N \qquad k := 0..N \qquad j := \sqrt{-1}$

Define smooth-edged lowpass and highpass functions (raised cosine).

$$LP(x1,x2,x) := \text{if}\left[x < x1, 1, \text{if}\left[x > x2, 0, \sqrt{0.5\cdot\left[1+\cos\left[\pi\cdot\left(\frac{x-x1}{x2-x1}\right)\right]\right]}\right]\right]$$

$$HP(x1,x2,x) := \text{if}\left[x < x1, 0, \text{if}\left[x > x2, 1, \sqrt{0.5\cdot\left[1-\cos\left[\pi\cdot\left(\frac{x-x1}{x2-x1}\right)\right]\right]}\right]\right]$$

$\theta_{i,k} := \text{angle}(i + 0.001 - a, k - a)$

$\rho_{i,k} := \sqrt{(i-a)^2 + (k-a)^2} \qquad c := \sqrt{\frac{8}{9}}$

$\text{sign}(x) := \text{if}(x < 0, -1, 1) \quad f1 := 0\cdot a \qquad f2 := \frac{5}{8}\cdot a$

Define the transfer functions of the one highpass, two lowpass and three bandpass filters.
The constants f1 and f2 control the steepness of the cutoffs, and c is a required fudge factor.
The LP(f3,f4) filter is not required, but it makes the kernels smaller.

$L0_{i,k} := LP\left(f2, a, \rho_{i,k}\right) \qquad H0_{i,k} := HP\left(f2, a, \rho_{i,k}\right) \qquad L1_{i,k} := LP\left(f1, \frac{a}{2}, \rho_{i,k}\right) \qquad f3 := \frac{6}{8}\cdot a \qquad f4 := 1.2\cdot a$

$$B1_{i,k} := c\cdot LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(f1, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k}\right)^2\cdot\text{sign}\left(\cos\left(\theta_{i,k}\right)\right)$$

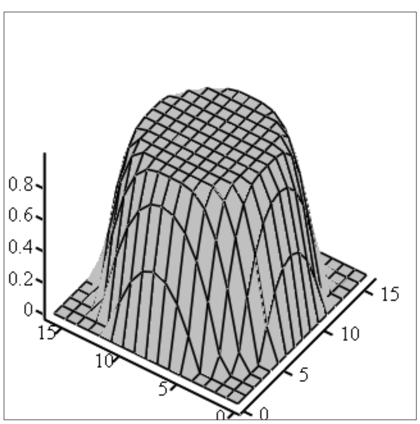
$$B2_{i,k} := c\cdot LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(f1, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k} - \frac{\pi}{3}\right)^2\cdot\text{sign}\left(\cos\left(\theta_{i,k} - \frac{\pi}{3}\right)\right)$$

$$B3_{i,k} := c\cdot LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(f1, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k} - \frac{2\cdot\pi}{3}\right)^2\cdot\text{sign}\left(\cos\left(\theta_{i,k} - \frac{2\cdot\pi}{3}\right)\right)$$

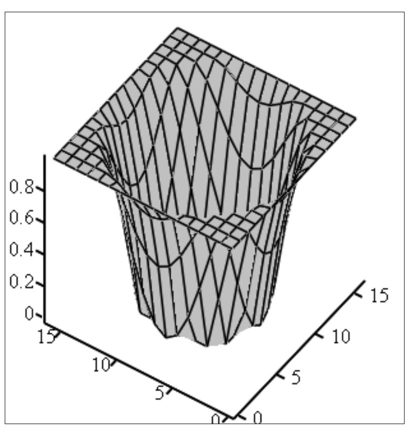
Compute functions to show that the constraints are satisfied.

$M1_{i,k} := \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2 + \left(B3_{i,k}\right)^2 \quad M2_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2 \quad M2_{N,N} := 0 \quad M3_{i,k} := M1_{i,k} + \left(L1_{i,k}\right)^2$

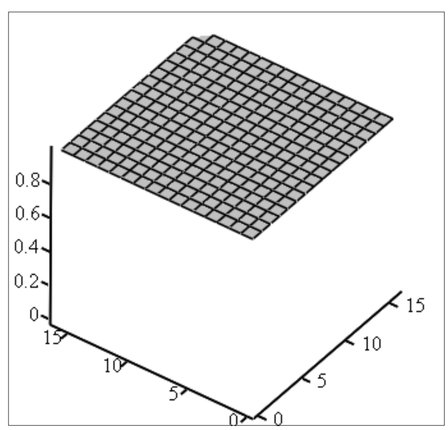
Plot $L_0(u,v)$, $H_0(u,v)$ and the sum of their squared magnitudes



L0

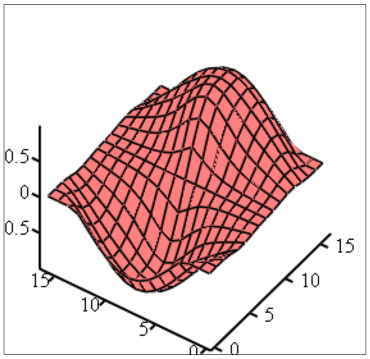


H0

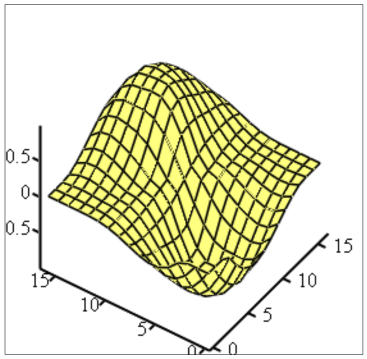


M2

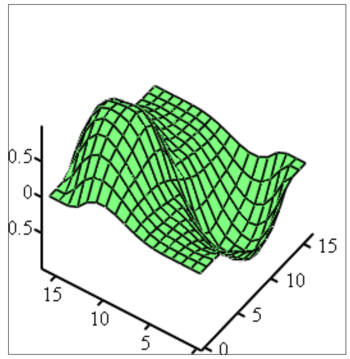
Plot the three steerable bandpass filters and their sum of squared magnitudes.



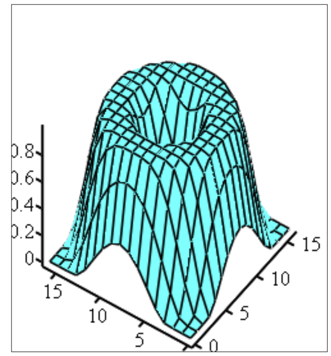
B1



B2

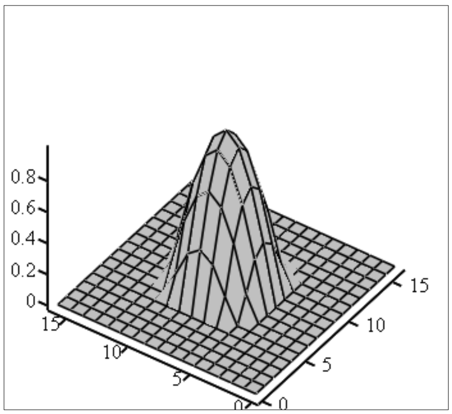


B3

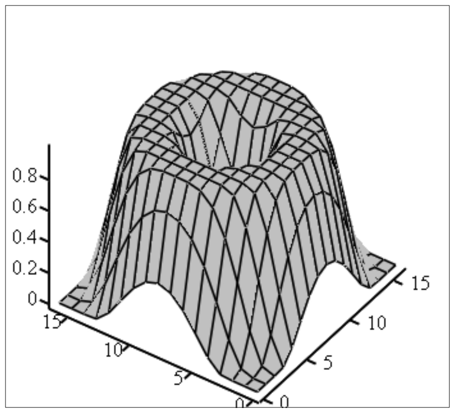


M1

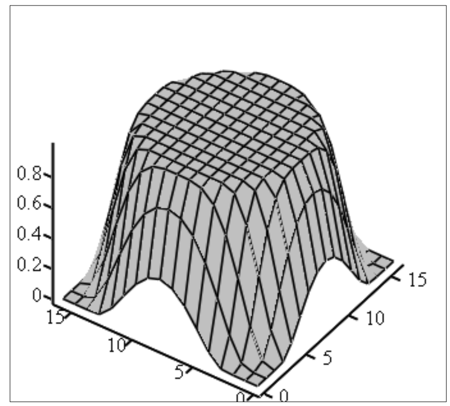
Plot $L_1(u,v)$ and the sum of its squared magnitude plus that of the three bandpass filters.



L1



M1

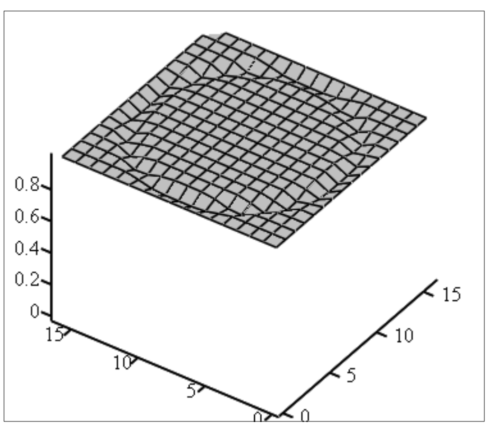


M3

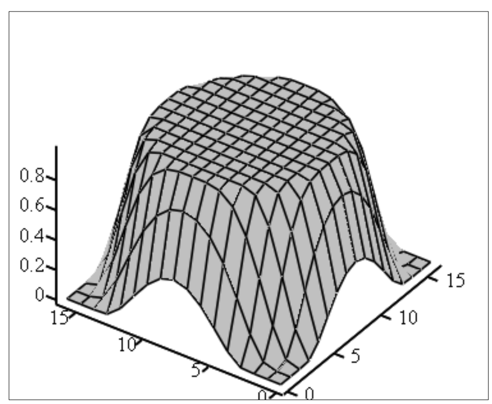
$$M4_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2 \cdot \left[\left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2 + \left(B3_{i,k}\right)^2\right] \quad M5_{i,k} := \left[\left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2 + \left(B3_{i,k}\right)\right.$$

$M4_{N,N} := 0 \quad M5_{N,N} := 0$

M4 = 1 is Simoncelli's constraint.



M4



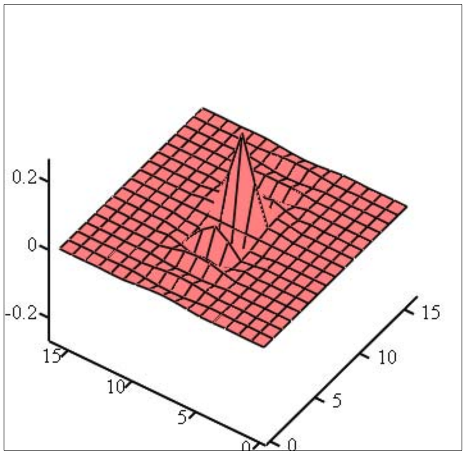
M5

Compute the convolution kernels using the centered DFT.

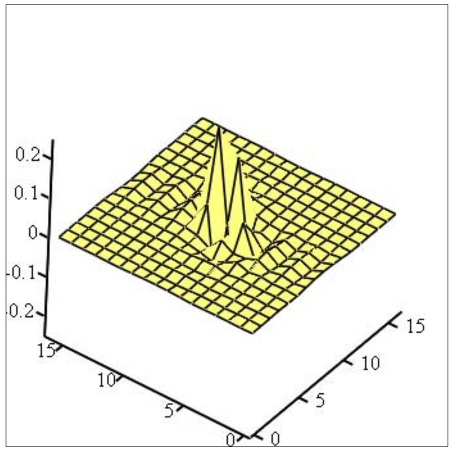
$$W_{i,k} := \frac{1}{N+1} \cdot \exp\left[-j \cdot 2 \cdot \pi \cdot (i-a) \cdot \frac{k-a}{N+1}\right]$$

$b1 := W \cdot (j \cdot B1) \cdot W \qquad b2 := W \cdot (j \cdot B2) \cdot W \qquad b3 := W \cdot (j \cdot B3) \cdot W \qquad h0 := W \cdot H0 \cdot W \qquad l0 := W \cdot L0 \cdot W \qquad l1 := W \cdot L1 \cdot W$

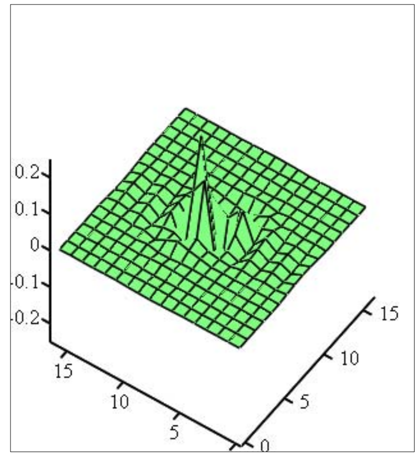
Plot the three bandpass filter impulse responses.



b1

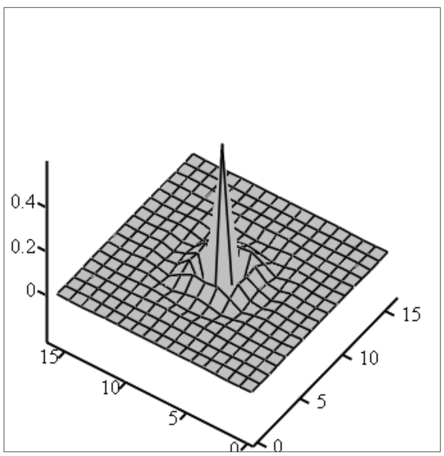


b2

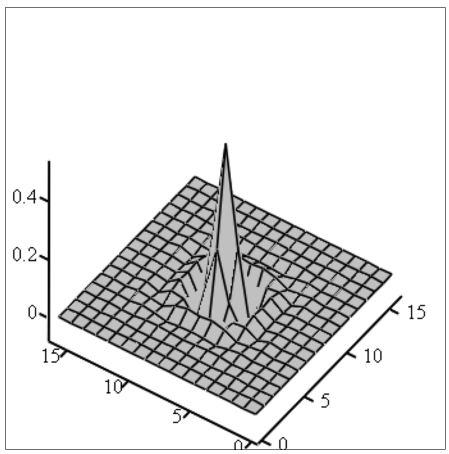


b3

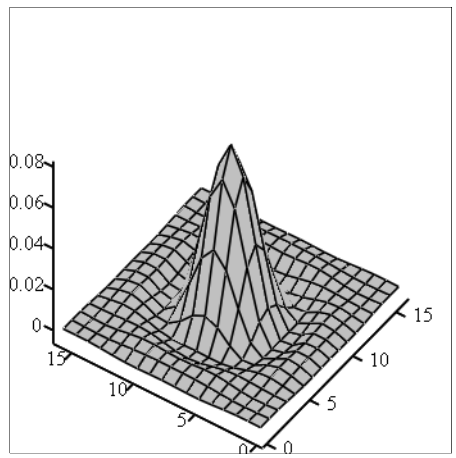
Plot the highpass and the two lowpass filter impulse responses.



h0



l0



l1

Round off to four digits and scale the values for writing out the kernel files. $b := 1$

$$K3\_L0_{i,k} := \frac{\text{floor}\left(10000\cdot\text{Re}\left(l0_{i,k}\right) + 0.5\right)}{b} \quad K3\_L1_{i,k} := \frac{\text{floor}\left(10000\cdot\text{Re}\left(l1_{i,k}\right) + 0.5\right)}{b} \quad K3\_H0_{i,k} := \frac{\text{floor}\left(10000\cdot\text{Re}\left(h0_{i,k}\right) + 0.5\right)}{b}$$

$$K3\_B1_{i,k} := \frac{\text{floor}\left(10000\cdot\text{Re}\left(b1_{i,k}\right) + 0.5\right)}{b} \quad K3\_B2_{i,k} := \frac{\text{floor}\left(10000\cdot\text{Re}\left(b2_{i,k}\right) + 0.5\right)}{b} \quad K3\_B3_{i,k} := \frac{\text{floor}\left(10000\cdot\text{Re}\left(b3_{i,k}\right) + 0.5\right)}{b}$$

$$\sum_i \sum_k K3\_L0_{i,k} = 9996 \quad \sum_i \sum_k K3\_L1_{i,k} = 10005 \quad \sum_i \sum_k K3\_H0_{i,k} = 0 \quad d := \begin{pmatrix} N+1 & N+1 \\ a & a \end{pmatrix}$$

$$\sum_i \sum_k K3\_B1_{i,k} = 0 \quad \sum_i \sum_k K3\_B2_{i,k} = 0 \quad \sum_i \sum_k K3\_B3_{i,k} = 0 \quad d = \begin{pmatrix} 17 & 17 \\ 8 & 8 \end{pmatrix}$$

1. E. P. Simoncelli, and W. T. Freeman, "The Steerable Pyramid: A Flexible Architecture for Multi-Scale Derivative Computation," *Proc. ICIP-95*: 444-447, 1995.
2. P. J. Burt, and E. H. Adelson, "The Laplacian Pyramid as a Compact Image Code," *IEEE Trans*. **C-31**:532-540, 1983.
3. E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger, "Shiftable Multiscale Transforms," *IEEE Trans*. **IT-38**(2):587-607, 1992.