Frequency Domain Steerable Pyramid Filter Design

First derivative, k = 2 filter **Ken Castleman** 5/28/98 StPyr_02.MCD

$N := 16 \quad a := \text{floor}\left(\frac{N}{2}\right) \quad i := 0..N \quad k := 0..N \quad j := \sqrt{-1}$

Define smooth-edged lowpass and highpass functions (raised cosine).

$$LP(x1,x2,x) := \text{if}\left[x < x1, 1, \text{if}\left[x > x2, 0, \sqrt{0.5\cdot\left[1 + \cos\left[\pi\cdot\left(\frac{x - x1}{x2 - x1}\right)\right]\right]}\right]\right]$$

$$HP(x1,x2,x) := \text{if}\left[x < x1, 0, \text{if}\left[x > x2, 1, \sqrt{0.5\cdot\left[1 - \cos\left[\pi\cdot\left(\frac{x - x1}{x2 - x1}\right)\right]\right]}\right]\right]$$

$$\theta_{i,k} := \text{angle}(i + 0.001 - a, k - a)$$

$$\rho_{i,k} := \sqrt{(i-a)^2 + (k-a)^2}$$

$f1 := 0\cdot a \quad f2 := \frac{5}{8}\cdot a \quad f3 := \frac{7}{8}\cdot a \quad f4 := 1.2\cdot a$

Define the transfer functions of the highpass, two lowpass and two bandpass filters.
The constants f1 and f2 control the steepness of the cutoffs; a is the folding frequency.
The LP(f3,f4,f) filter is not required for invertibility, but it makes the bandpass kernels smaller.

$$B1_{i,k} := LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(f1, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k}\right) \qquad B2_{i,k} := LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(f1, \frac{a}{2}, \rho_{i,k}\right)\cdot\cos\left(\theta_{i,k} - \frac{\pi}{2}\right)$$

$$L1_{i,k} := LP\left(f1, \frac{a}{2}, \rho_{i,k}\right) \qquad L0_{i,k} := LP\left(f2, a, \rho_{i,k}\right) \qquad H0_{i,k} := HP\left(f2, a, \rho_{i,k}\right)$$

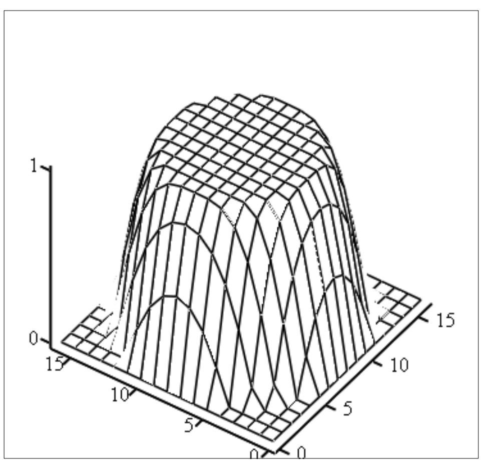
Compute functions to show that the constraints are satisfied.

$$M1_{i,k} := \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2 \qquad M2_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2 \qquad M3_{i,k} := \left[\left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2\right]$$

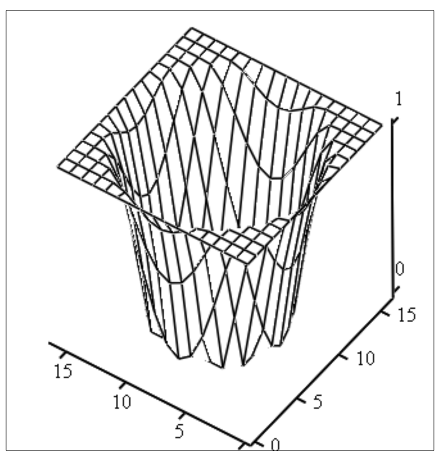
$$M4_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2\cdot\left[\left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2 + \left(B2_{i,k}\right)^2\right]$$

M4 = 1 is Simoncelli's constraint.

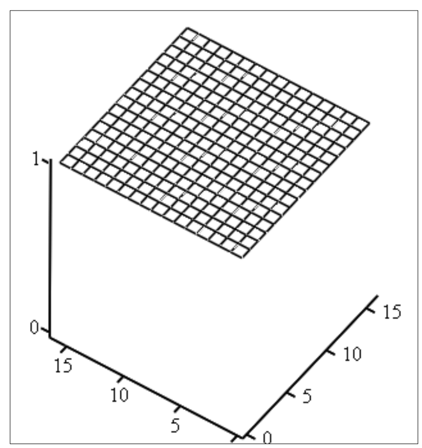
Plot $L_0(u,v)$, $H_0(u,v)$ and the sum of their squared magnitudes



L0

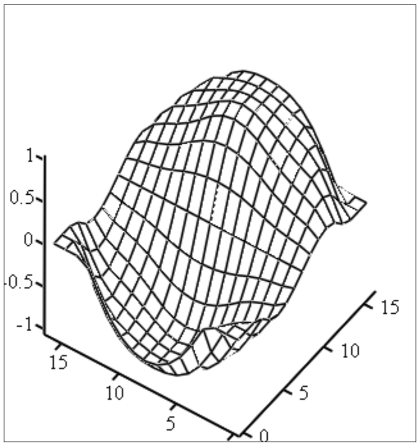


H0

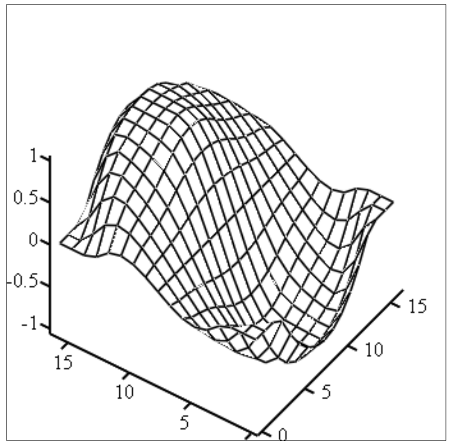


M2

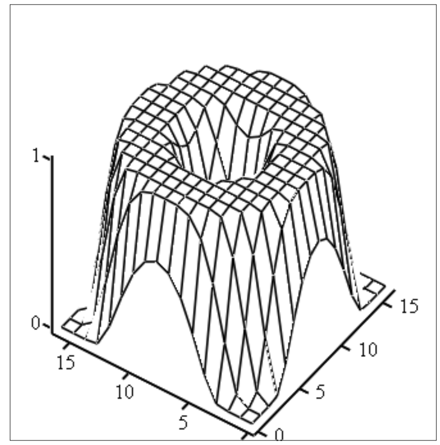
Plot the two steerable bandpass filters and their sum of squared magnitudes.



B1

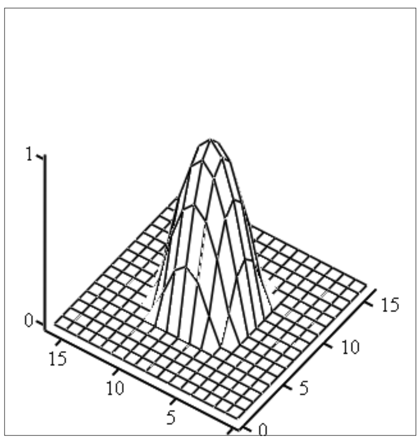


B2

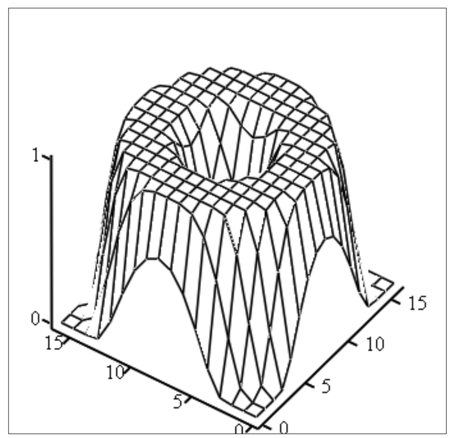


M1

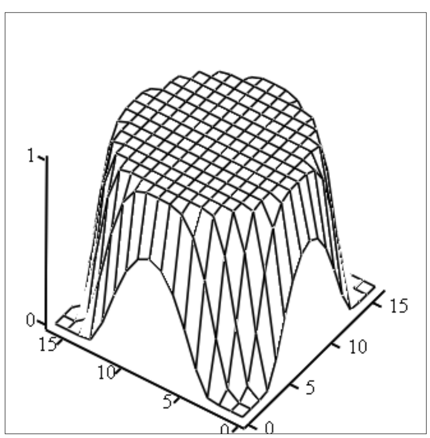
Plot $L_1(u,v)$, the sum of the bandpass squared magnitudes, their sum, and the overall transfer function.



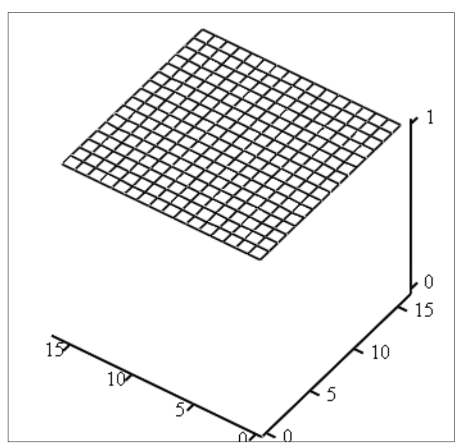
L1



M1



M3

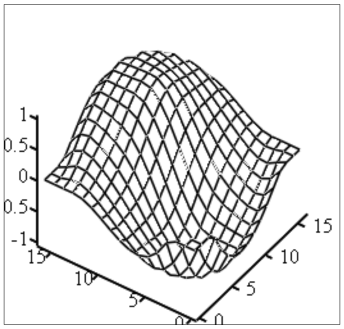


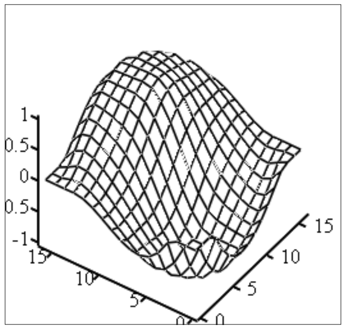
M4

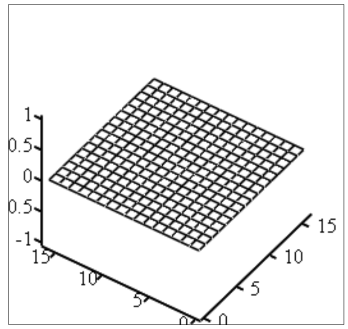
Demonstrate steerability - generate and synthesize 60 degree filters and compare them.

$$B3_{i,k} := LP(f3, f4, \rho_{i,k}) \cdot HP\left(f1, \frac{a}{2}, \rho_{i,k}\right) \cdot \cos\left(\theta_{i,k} - \frac{\pi}{3}\right)$$

$$B4 := \frac{1}{2} \cdot B1 + \frac{\sqrt{3}}{2} \cdot B2$$







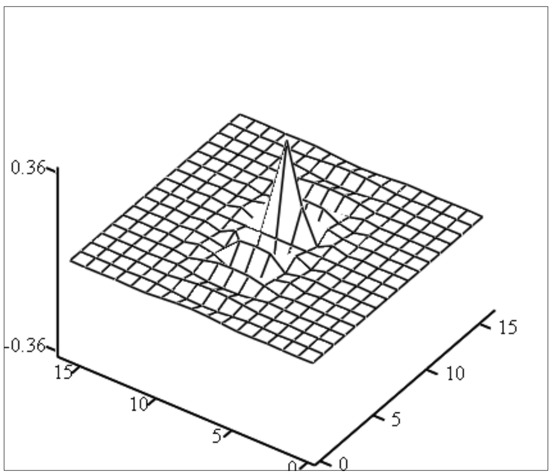
B3 B4 B4 − B3

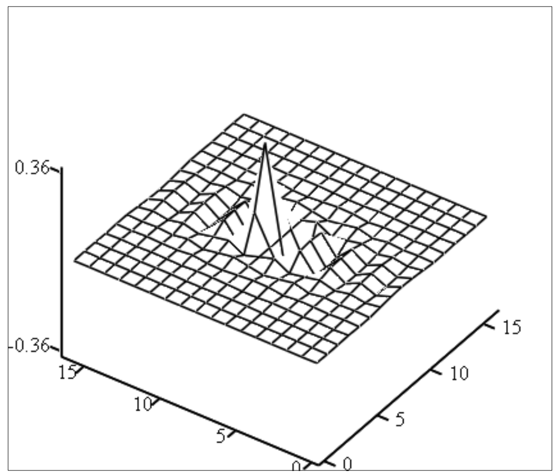
Compute the convolution kernels using the centered DFT.

$$W_{i,k} := \frac{1}{N+1}\cdot \exp\left[-j\cdot 2\cdot \pi\cdot (i-a)\cdot \frac{k-a}{N+1}\right]$$

$b1 := W\cdot(j\cdot B1)\cdot W$ $\quad b2 := W\cdot(j\cdot B2)\cdot W$ $\quad l0 := W\cdot L0\cdot W$ $\quad l1 := W\cdot L1\cdot W$ $\quad h0 := W\cdot H0\cdot W$

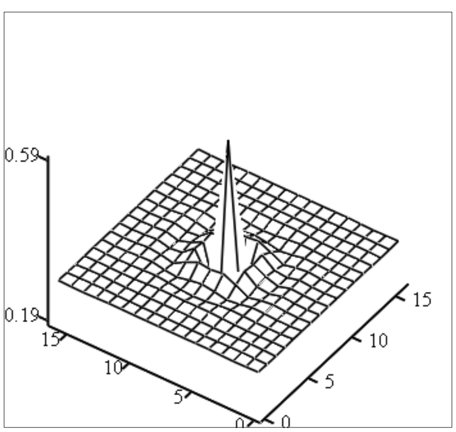
Plot the two bandpass filter impulse responses.

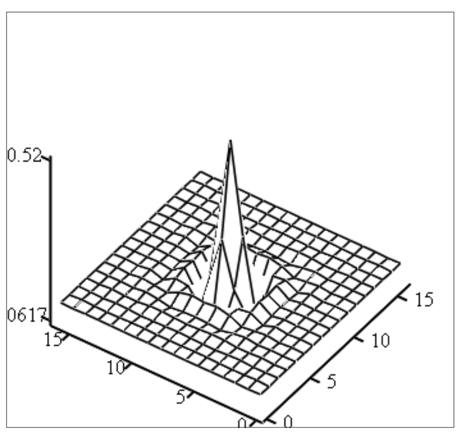


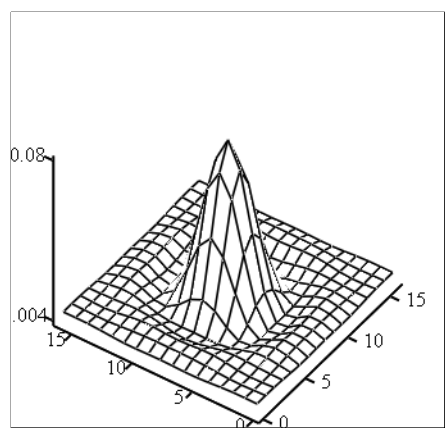


b1 b2

Plot the highpass and the two lowpass filter impulse responses.







h0 l0 l1

$$\sum_i \sum_k l0_{i,k} = 1 \qquad \sum_i \sum_k l1_{i,k} = 1 \qquad \sum_i \sum_k h0_{i,k} = 0 \qquad \sum_i \sum_k b1_{i,k} = 0 \qquad \sum_i \sum_k b2_{i,k} = 0$$

$$d := \begin{pmatrix} N+1 & N+1 \\ a & a \end{pmatrix} \qquad d = \begin{pmatrix} 17 & 17 \\ 8 & 8 \end{pmatrix}$$

Round off to four digits and scale the values for writing kernel files.

$b := 1$

$$K2\_L0_{i,k} := \frac{floor(10000 \cdot Re(l0_{i,k}) + 0.5)}{b} \qquad K2\_L1_{i,k} := \frac{floor(10000 \cdot Re(l1_{i,k}) + 0.5)}{b}$$

$$K2\_H0_{i,k} := \frac{floor(10000 \cdot Re(h0_{i,k}) + 0.5)}{b}$$

$$K2\_B1_{i,k} := \frac{floor(10000 \cdot Re(b1_{i,k}) + 0.5)}{b} \qquad K2\_B2_{i,k} := \frac{floor(10000 \cdot Re(b2_{i,k}) + 0.5)}{b}$$

$$\sum_i \sum_k K2\_L0_{i,k} = 9996 \quad \sum_i \sum_k K2\_L1_{i,k} = 10005 \quad \sum_i \sum_k K2\_H0_{i,k} = 0 \quad \sum_i \sum_k K2\_B1_{i,k} = 0 \quad \sum_i \sum_k K2\_B2_{i,k} = 0$$

Write kernel files for input to the WiT 2-D convolution operator (Use b = 10,000 for unscaled kernels).

$$WRITEPRN(K2\_L0_{arr}) := d \qquad APPENDPRN(K2\_L0_{arr}) := K2\_L0$$

$$WRITEPRN(K2\_L1_{arr}) := d \qquad APPENDPRN(K2\_L1_{arr}) := K2\_L1$$

$$WRITEPRN(K2\_H0_{arr}) := d \qquad APPENDPRN(K2\_H0_{arr}) := K2\_H0$$

$$WRITEPRN(K2\_B1_{arr}) := d \qquad APPENDPRN(K2\_B1_{arr}) := K2\_B1$$

$$WRITEPRN(K2\_B2_{arr}) := d \qquad APPENDPRN(K2\_B2_{arr}) := K2\_B2$$

1. E. P. Simoncelli, and W. T. Freeman, "The Steerable Pyramid: A Flexible Architecture for Multi-Scale Derivative Computation," *Proc. ICIP-95*: 444-447, 1995.
2. P. J. Burt, and E. H. Adelson, "The Laplacian Pyramid as a Compact Image Code," *IEEE Trans*. **C-31**:532-540, 1983.
3. E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger, "Shiftable Multiscale Transforms," *IEEE Trans*. **IT-38**(2):587-607, 1992.
4. W. T. Freeman and E. H. Adelson, "The Design and Use of Steerable Filters," *IEEE Trans* **PAMI-13**(9):891-906, 1991.

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