

Frequency Domain Steerable Pyramid Filter Design

Non-directional, $k = 1$ filter

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StPyr_01.MCD

$$N := 16 \quad a := \text{floor}\left(\frac{N}{2}\right) \quad i := 0..N \quad k := 0..N \quad j := \sqrt{-1}$$

Define lowpass and highpass functions (based on the raised cosine) with a smooth transition from x_1 to x_2 .

$$\text{LP}(x_1, x_2, x) := \text{if} \left[x < x_1, 1, \text{if} \left[x > x_2, 0, \sqrt{0.5 \cdot \left[1 + \cos \left[\pi \cdot \left(\frac{x - x_1}{x_2 - x_1} \right) \right] \right]} \right] \right]$$

$$\rho_{i,k} := \sqrt{(i - a)^2 + (k - a)^2}$$

$$\text{HP}(x_1, x_2, x) := \text{if} \left[x < x_1, 0, \text{if} \left[x > x_2, 1, \sqrt{0.5 \cdot \left[1 - \cos \left[\pi \cdot \left(\frac{x - x_1}{x_2 - x_1} \right) \right] \right]} \right] \right]$$

$$f_1 := 0 \cdot a \quad f_2 := \frac{5}{8} \cdot a \quad f_3 := \frac{12}{8} \cdot a \quad f_4 := 1.2 \cdot a$$

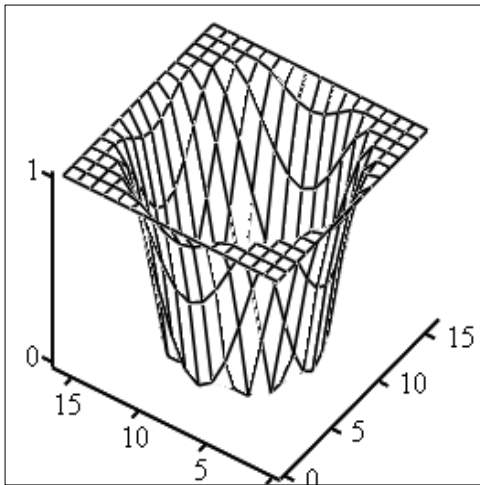
Define transfer functions for the highpass, bandpass and two lowpass filters.

The constants f_1 and f_2 control the steepness of the cutoffs; a is the folding frequency.

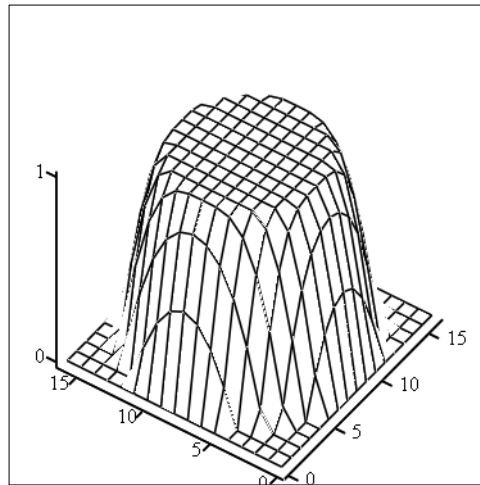
The $\text{LP}(f_3, f_4, f)$ filter is not required for invertibility, and using it makes the bandpass kernel larger.

$$H_{0,i,k} := \text{HP}(f_2, a, \rho_{i,k}) \quad L_{0,i,k} := \text{LP}(f_2, a, \rho_{i,k}) \quad B_{1,i,k} := \text{LP}(f_3, f_4, \rho_{i,k}) \cdot \text{HP}\left(f_1, \frac{a}{2}, \rho_{i,k}\right) \quad L_{1,i,k} := \text{LP}\left(f_1, \frac{a}{2}, \rho_{i,k}\right)$$

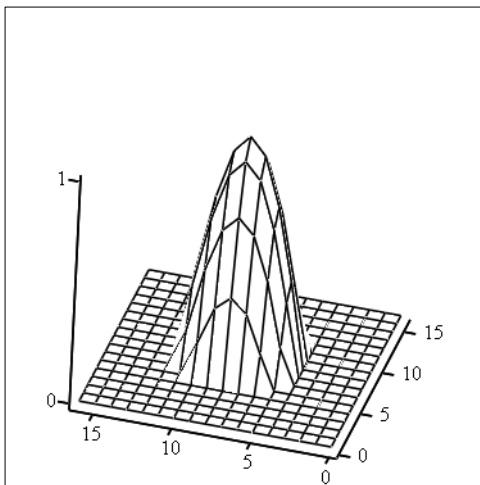
Plot the transfer functions $L_0(u,v)$, $H_0(u,v)$, $L_1(u,v)$, and $B_1(u,v)$



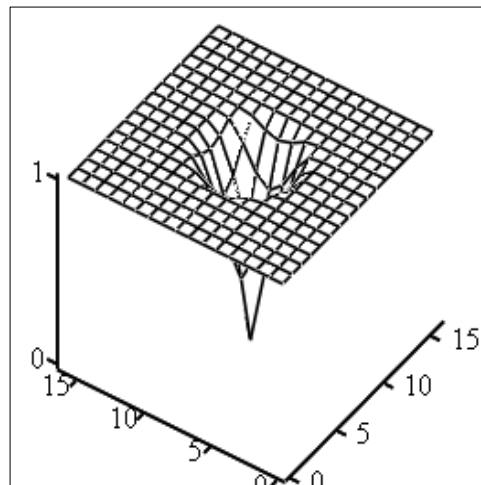
H0



L0



L1



B1

Simoncelli's invertibility constraint is: $(H_0)^2 + (L_0)^2 \cdot [(L_1)^2 + (B_1)^2] = 1$

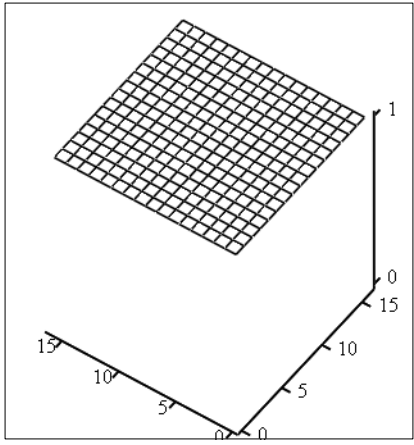
Castleman's sufficient conditions are: $(H_0)^2 + (L_0)^2 = 1$ $(L_1)^2 + (B_1)^2 = 1$

Compute functions to determine if the constraints are satisfied.

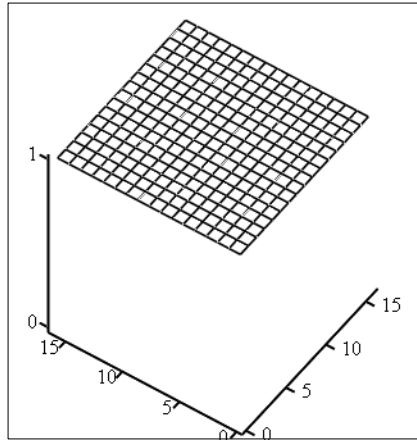
$$M2_{i,k} := (H0_{i,k})^2 + (L0_{i,k})^2 \quad M3_{i,k} := (L1_{i,k})^2 + (B1_{i,k})^2 \quad M4_{i,k} := (H0_{i,k})^2 + (L0_{i,k})^2 \cdot [(L1_{i,k})^2 + (B1_{i,k})^2]$$

Plot the constraints.

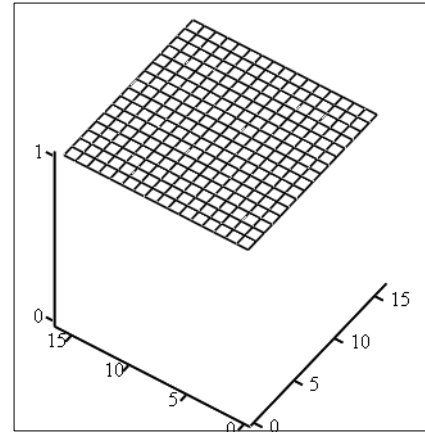
M4 = 1 is Simoncelli's constraint.



M2



M3



M4

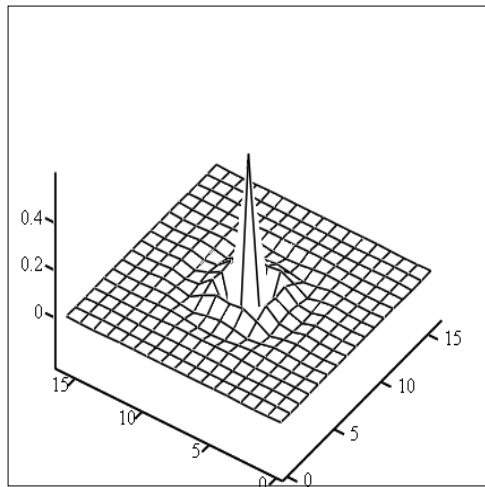
The Convolution Kernels

Compute the convolution kernels using the centered DFT.

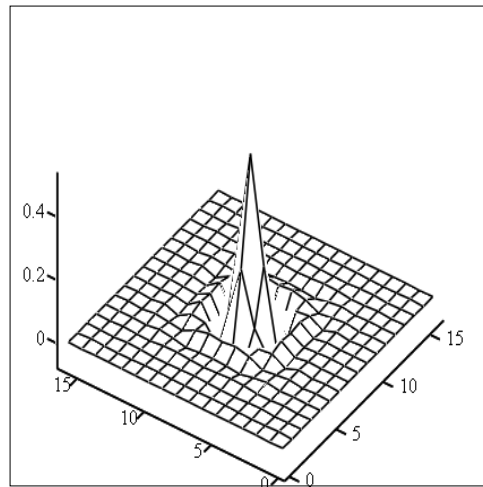
$$W_{i,k} := \frac{1}{N+1} \cdot \exp\left[-j \cdot 2 \cdot \pi \cdot (i-a) \cdot \frac{k-a}{N+1}\right]$$

$$b1 := W \cdot B1 \cdot W \quad l0 := W \cdot L0 \cdot W \quad l1 := W \cdot L1 \cdot W \quad h0 := W \cdot H0 \cdot W$$

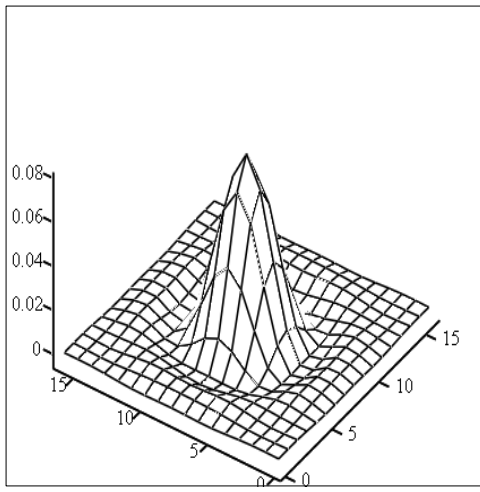
Plot the bandpass, highpass and the two lowpass filter impulse responses.



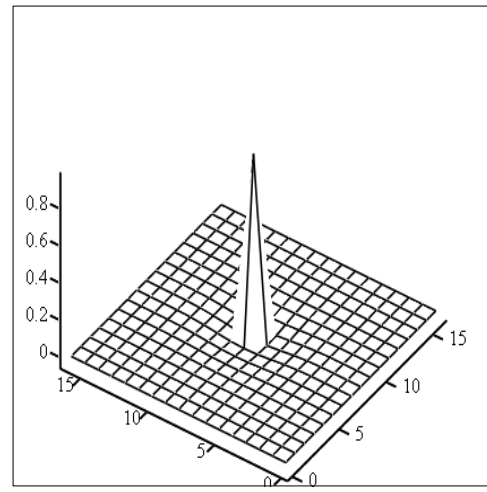
h0



l0



l1



b1

$$10000 \cdot b1 = \begin{pmatrix} -6 & -6 & -5 & -5 & -6 & -8 & -9 & -9 & -9 & -9 & -9 & -8 & -6 & -5 & -5 & -6 & -6 \\ -6 & -6 & -6 & -7 & -8 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -8 & -7 & -6 & -6 & -6 \\ -5 & -6 & -8 & -9 & -10 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -10 & -9 & -8 & -6 & -5 \\ -5 & -7 & -9 & -10 & -9 & -9 & -11 & -14 & -15 & -14 & -11 & -9 & -9 & -10 & -9 & -7 & -5 \\ -6 & -8 & -10 & -9 & -9 & -15 & -27 & -40 & -45 & -40 & -27 & -15 & -9 & -9 & -10 & -8 & -6 \\ -8 & -9 & -9 & -9 & -15 & -34 & -67 & -100 & -113 & -100 & -67 & -34 & -15 & -9 & -9 & -9 & -8 \\ -9 & -9 & -9 & -11 & -27 & -67 & -129 & -186 & -209 & -186 & -129 & -67 & -27 & -11 & -9 & -9 & -9 \\ -9 & -9 & -9 & -14 & -40 & -100 & -186 & -265 & -297 & -265 & -186 & -100 & -40 & -14 & -9 & -9 & -9 \\ -9 & -9 & -9 & -15 & -45 & -113 & -209 & -297 & 9667 & -297 & -209 & -113 & -45 & -15 & -9 & -9 & -9 \\ -9 & -9 & -9 & -14 & -40 & -100 & -186 & -265 & -297 & -265 & -186 & -100 & -40 & -14 & -9 & -9 & -9 \\ -9 & -9 & -9 & -11 & -27 & -67 & -129 & -186 & -209 & -186 & -129 & -67 & -27 & -11 & -9 & -9 & -9 \\ -8 & -9 & -9 & -9 & -15 & -34 & -67 & -100 & -113 & -100 & -67 & -34 & -15 & -9 & -9 & -9 & -8 \\ -6 & -8 & -10 & -9 & -9 & -15 & -27 & -40 & -45 & -40 & -27 & -15 & -9 & -9 & -10 & -8 & -6 \\ -5 & -7 & -9 & -10 & -9 & -9 & -11 & -14 & -15 & -14 & -11 & -9 & -9 & -10 & -9 & -7 & -5 \\ -5 & -6 & -8 & -9 & -10 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -10 & -9 & -8 & -6 & -5 \\ -6 & -6 & -6 & -7 & -8 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -8 & -7 & -6 & -6 & -6 \\ -6 & -6 & -5 & -5 & -6 & -8 & -9 & -9 & -9 & -9 & -9 & -8 & -6 & -5 & -5 & -6 & -6 \end{pmatrix}$$

$$\sum_i \sum_k 10_{i,k} = 1 \quad \sum_i \sum_k 11_{i,k} = 1 \quad \sum_i \sum_k h0_{i,k} = 0 \quad \sum_i \sum_k b1_{i,k} = 0 \quad d := \begin{pmatrix} N+1 & N+1 \\ a & a \end{pmatrix} \quad d = \begin{pmatrix} 17 & 17 \\ 8 & 8 \end{pmatrix}$$

Round off to four digits and scale the values for writing kernel files.

$$b := 1$$

$$K1_L0_{i,k} := \frac{\text{floor}(10000 \cdot \text{Re}(10_{i,k}) + 0.5)}{b} \quad K1_L1_{i,k} := \frac{\text{floor}(10000 \cdot \text{Re}(11_{i,k}) + 0.5)}{b} \quad K1_H0_{i,k} := \frac{\text{floor}(10000 \cdot \text{Re}(h0_{i,k}) + 0.5)}{b}$$

$$K1_B1_{i,k} := \frac{\text{floor}(10000 \cdot \text{Re}(b1_{i,k}) + 0.5)}{b}$$

$$\sum_i \sum_k K1_L0_{i,k} = 9996 \quad \sum_i \sum_k K1_L1_{i,k} = 10005 \quad \sum_i \sum_k K1_H0_{i,k} = 0 \quad \sum_i \sum_k K1_B1_{i,k} = 15$$

Write kernel files for input to the WiT 2-D convolution operator (Use b = 10,000 for unscaled kernels).

```
WRITEPRN("K1_L0.arr") := d   APPENDPRN("K1_L0.arr") := K1_L0  
WRITEPRN("K1_L1.arr") := d   APPENDPRN("K1_L1.arr") := K1_L1  
WRITEPRN("K1_H0.arr") := d   APPENDPRN("K1_H0.arr") := K1_H0  
WRITEPRN("K1_B1.arr") := d   APPENDPRN("K1_B1.arr") := K1_B1
```

1. E. P. Simoncelli, and W. T. Freeman, "The Steerable Pyramid: A Flexible Architecture for Multi-Scale Derivative Computation," *Proc. ICIP-95*: 444-447, 1995.
2. P. J. Burt, and E. H. Adelson, "The Laplacian Pyramid as a Compact Image Code," *IEEE Trans. C-31*:532-540, 1983.
3. E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger, "Shiftable Multiscale Transforms," *IEEE Trans. IT-38*(2):587-607, 1992.
4. K. R. Castleman, M.Schulze, and Q. Wu, "Simplified Design of Steerable Pyramid Filters," *Proc. ISCAS '98*, June 3, 1998.

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