Frequency Domain Steerable Pyramid Filter Design

Non-directional, k = 1 filter **Ken Castleman** 4/27/98 StPyr_01.MCD

$N := 16 \qquad a := \mathrm{floor}\left(\frac{N}{2}\right) \qquad i := 0 .. N \qquad k := 0 .. N \qquad j := \sqrt{-1}$

Define lowpass and highpass functions (based on the raised cosine) with a smooth transition from x1 to x2.

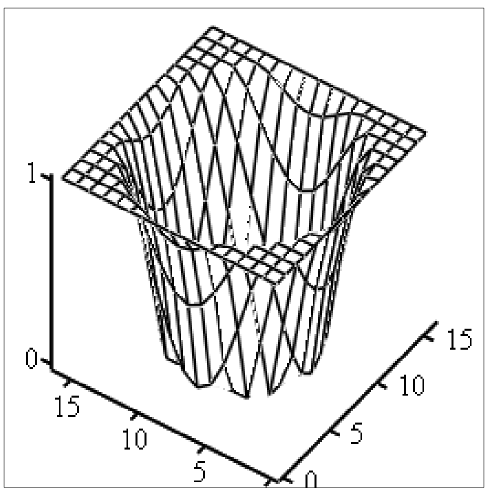
$$LP(x1, x2, x) := \mathrm{if}\left[x < x1, 1, \mathrm{if}\left[x > x2, 0, \sqrt{0.5\cdot\left[1 + \cos\left[\pi\cdot\left(\frac{x - x1}{x2 - x1}\right)\right]\right]}\right]\right] \qquad \rho_{i,k} := \sqrt{(i-a)^2 + (k-a)^2}$$

$$HP(x1, x2, x) := \mathrm{if}\left[x < x1, 0, \mathrm{if}\left[x > x2, 1, \sqrt{0.5\cdot\left[1 - \cos\left[\pi\cdot\left(\frac{x - x1}{x2 - x1}\right)\right]\right]}\right]\right] \qquad f1 := 0\cdot a \quad f2 := \frac{5}{8}\cdot a \quad f3 := \frac{12}{8}\cdot a \quad f4 := 1.2\cdot a$$

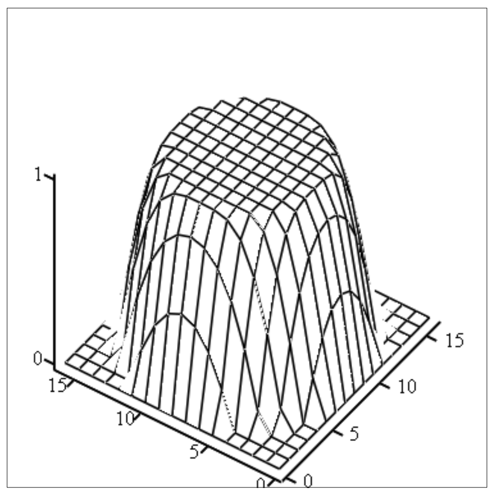
Define transfer functions for the highpass, bandpass and two lowpass filters.
The constants f1 and f2 control the steepness of the cutoffs; a is the folding frequency.
The LP(f3,f4,f) filter is not required for invertibility, and using it makes the bandpass kernel larger.

$$H0_{i,k} := HP\left(f2, a, \rho_{i,k}\right) \qquad L0_{i,k} := LP\left(f2, a, \rho_{i,k}\right) \qquad B1_{i,k} := LP\left(f3, f4, \rho_{i,k}\right)\cdot HP\left(f1, \frac{a}{2}, \rho_{i,k}\right) \qquad L1_{i,k} := LP\left(f1, \frac{a}{2}, \rho_{i,k}\right)$$

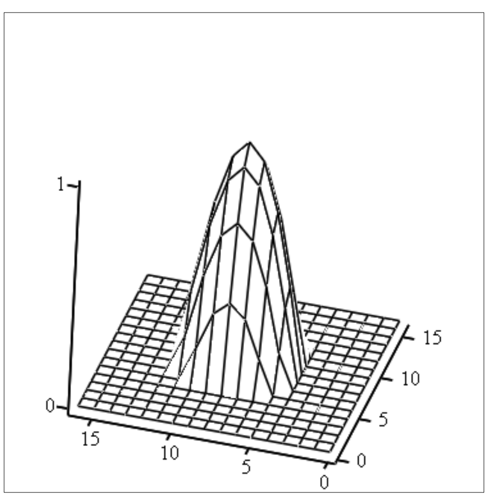
Plot the ransfer functions $L_0(u,v)$, $H_0(u,v)$, $L_1(u,v)$, and $B_1(u,v)$



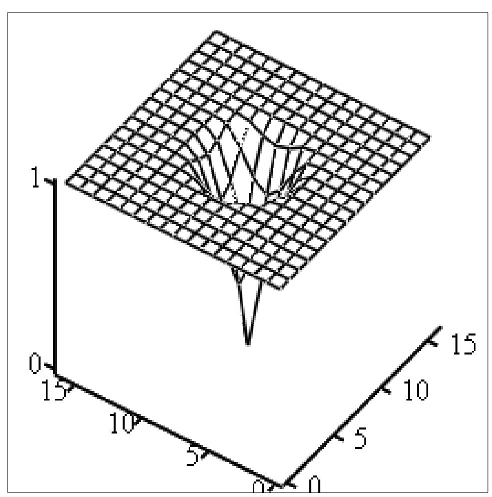
H0



L0



L1



B1

Simoncelli's invertibility constraint is: $\left(H_0\right)^2 + \left(L_0\right)^2 \cdot \left[\left(L_1\right)^2 + \left(B_1\right)^2\right] = 1$

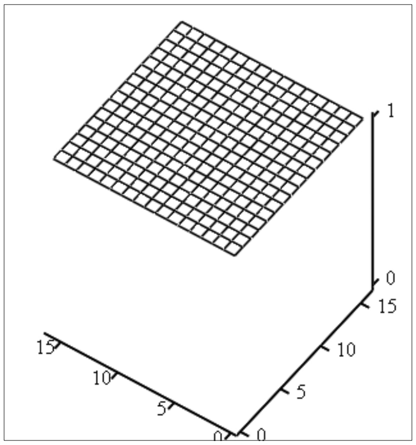
Castleman's sufficient conditions are: $\left(H_0\right)^2 + \left(L_0\right)^2 = 1$ $\quad \left(L_1\right)^2 + \left(B_1\right)^2 = 1$

Compute functions to determine if the constraints are satisfied.

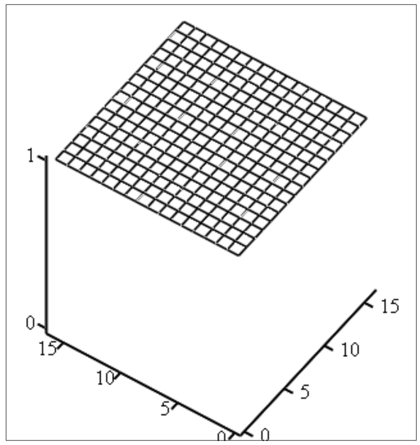
$M2_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2 \qquad M3_{i,k} := \left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2 \qquad M4_{i,k} := \left(H0_{i,k}\right)^2 + \left(L0_{i,k}\right)^2 \cdot \left[\left(L1_{i,k}\right)^2 + \left(B1_{i,k}\right)^2\right]$

Plot the constraints.

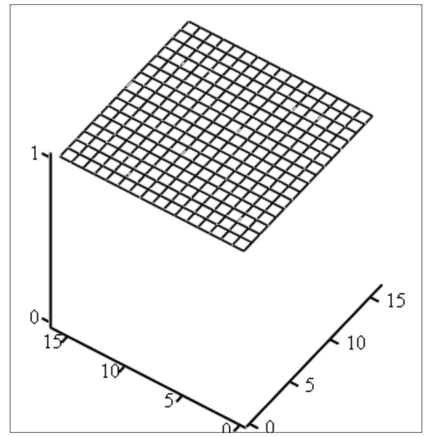
M4 = 1 is Simoncelli's constraint.



M2



M3



M4

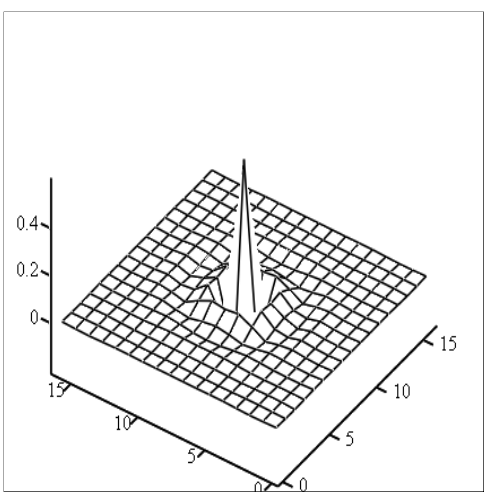
The Convolution Kernels

Compute the convolution kernels using the centered DFT.

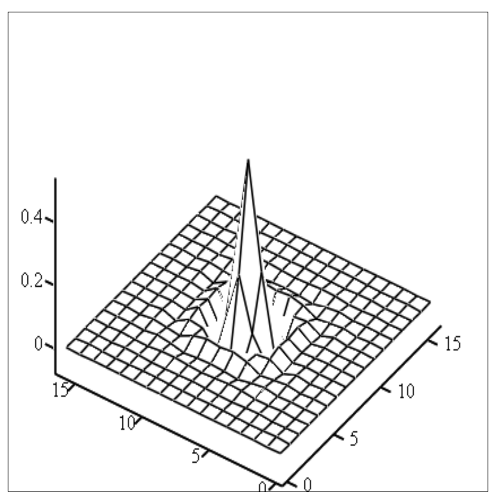
$$W_{i,k} := \frac{1}{N+1} \cdot \exp\left[-j \cdot 2 \cdot \pi \cdot (i-a) \cdot \frac{k-a}{N+1}\right]$$

$b1 := W \cdot B1 \cdot W \qquad l0 := W \cdot L0 \cdot W \qquad l1 := W \cdot L1 \cdot W \qquad h0 := W \cdot H0 \cdot W$

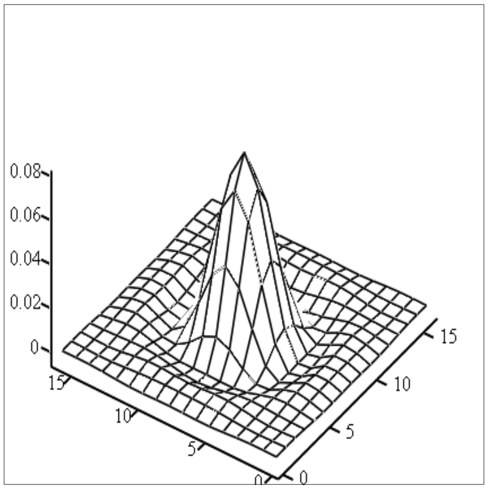
Plot the bandpass, highpass and the two lowpass filter impulse responses.



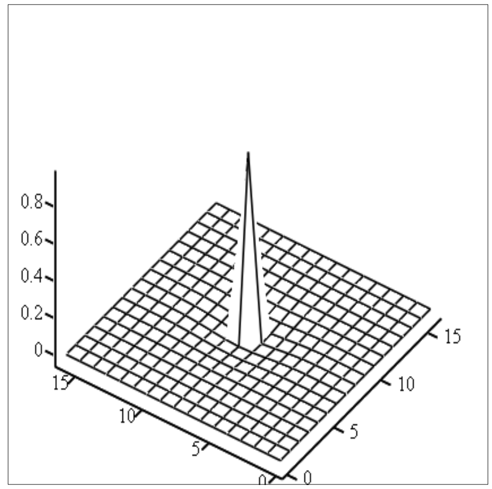
h0



l0



l1



b1

$$10000 \cdot b1 = \begin{pmatrix} -6 & -6 & -5 & -5 & -6 & -8 & -9 & -9 & -9 & -9 & -9 & -8 & -6 & -5 & -5 & -6 & -6 \\ -6 & -6 & -6 & -7 & -8 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -8 & -7 & -6 & -6 & -6 \\ -5 & -6 & -8 & -9 & -10 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -10 & -9 & -8 & -6 & -5 \\ -5 & -7 & -9 & -10 & -9 & -9 & -11 & -14 & -15 & -14 & -11 & -9 & -9 & -10 & -9 & -7 & -5 \\ -6 & -8 & -10 & -9 & -9 & -15 & -27 & -40 & -45 & -40 & -27 & -15 & -9 & -9 & -10 & -8 & -6 \\ -8 & -9 & -9 & -9 & -15 & -34 & -67 & -100 & -113 & -100 & -67 & -34 & -15 & -9 & -9 & -9 & -8 \\ -9 & -9 & -9 & -11 & -27 & -67 & -129 & -186 & -209 & -186 & -129 & -67 & -27 & -11 & -9 & -9 & -9 \\ -9 & -9 & -9 & -14 & -40 & -100 & -186 & -265 & -297 & -265 & -186 & -100 & -40 & -14 & -9 & -9 & -9 \\ -9 & -9 & -9 & -15 & -45 & -113 & -209 & -297 & 9667 & -297 & -209 & -113 & -45 & -15 & -9 & -9 & -9 \\ -9 & -9 & -9 & -14 & -40 & -100 & -186 & -265 & -297 & -265 & -186 & -100 & -40 & -14 & -9 & -9 & -9 \\ -9 & -9 & -9 & -11 & -27 & -67 & -129 & -186 & -209 & -186 & -129 & -67 & -27 & -11 & -9 & -9 & -9 \\ -8 & -9 & -9 & -9 & -15 & -34 & -67 & -100 & -113 & -100 & -67 & -34 & -15 & -9 & -9 & -9 & -8 \\ -6 & -8 & -10 & -9 & -9 & -15 & -27 & -40 & -45 & -40 & -27 & -15 & -9 & -9 & -10 & -8 & -6 \\ -5 & -7 & -9 & -10 & -9 & -9 & -11 & -14 & -15 & -14 & -11 & -9 & -9 & -10 & -9 & -7 & -5 \\ -5 & -6 & -8 & -9 & -10 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -10 & -9 & -8 & -6 & -5 \\ -6 & -6 & -6 & -7 & -8 & -9 & -9 & -9 & -9 & -9 & -9 & -9 & -8 & -7 & -6 & -6 & -6 \\ -6 & -6 & -5 & -5 & -6 & -8 & -9 & -9 & -9 & -9 & -9 & -8 & -6 & -5 & -5 & -6 & -6 \end{pmatrix}$$

$$\sum_i \sum_k l0_{i,k} = 1 \quad \sum_i \sum_k l1_{i,k} = 1 \quad \sum_i \sum_k h0_{i,k} = 0 \quad \sum_i \sum_k b1_{i,k} = 0 \quad d := \begin{pmatrix} N+1 & N+1 \\ a & a \end{pmatrix} \quad d = \begin{pmatrix} 17 & 17 \\ 8 & 8 \end{pmatrix}$$

Round off to four digits and scale the values for writing kernel files.

$$b := 1$$

$$K1\_L0_{i,k} := \frac{\mathrm{floor}\left(10000 \cdot \mathrm{Re}\left(l0_{i,k}\right) + 0.5\right)}{b} \quad K1\_L1_{i,k} := \frac{\mathrm{floor}\left(10000 \cdot \mathrm{Re}\left(l1_{i,k}\right) + 0.5\right)}{b} \quad K1\_H0_{i,k} := \frac{\mathrm{floor}\left(10000 \cdot \mathrm{Re}\left(h0_{i,k}\right) + 0.5\right)}{b}$$

$$K1\_B1_{i,k} := \frac{\mathrm{floor}\left(10000 \cdot \mathrm{Re}\left(b1_{i,k}\right) + 0.5\right)}{b}$$

$$\sum_i \sum_k K1\_L0_{i,k} = 9996 \quad \sum_i \sum_k K1\_L1_{i,k} = 10005 \quad \sum_i \sum_k K1\_H0_{i,k} = 0 \quad \sum_i \sum_k K1\_B1_{i,k} = 15$$

Write kernel files for input to the WiT 2-D convolution operator (Use b = 10,000 for unscaled kernels).

WRITEPRN("K1_L0.arr") := d   APPENDPRN("K1_L0.arr") := K1_L0

WRITEPRN("K1_L1.arr") := d   APPENDPRN("K1_L1.arr") := K1_L1

WRITEPRN("K1_H0.arr") := d   APPENDPRN("K1_H0.arr") := K1_H0

WRITEPRN("K1_B1.arr") := d   APPENDPRN("K1_B1.arr") := K1_B1

1. E. P. Simoncelli, and W. T. Freeman, "The Steerable Pyramid: A Flexible Architecture for Multi-Scale Derivative Computation," *Proc. ICIP-95*: 444-447, 1995.
2. P. J. Burt, and E. H. Adelson, "The Laplacian Pyramid as a Compact Image Code," *IEEE Trans*. **C-31**:532-540, 1983.
3. E. P. Simoncelli, W. T. Freeman, E. H. Adelson, and D. J. Heeger, "Shiftable Multiscale Transforms," *IEEE Trans*. **IT-38**(2):587-607, 1992.
4. K. R. Castleman, M.Schulze, and Q. Wu, "Simplified Design of Steerable Pyramid Filters," *Proc. ISCAS '98,* June 3, 1998.

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