2-D Wavelet Transform

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Mallat's Algorithm in 2-D

haar_01.mcd

$N := 7 \quad i := 0..N \quad j := 0..N \quad N1 := 3.5 \quad b \equiv 1.2 \quad a \equiv \frac{1}{2} \quad i2 := 0..\frac{N-1}{2} \quad j2 := 0..\frac{N-1}{2} \quad m := 0..1 \quad n := 0..1$

Use the 2-D Harr basis

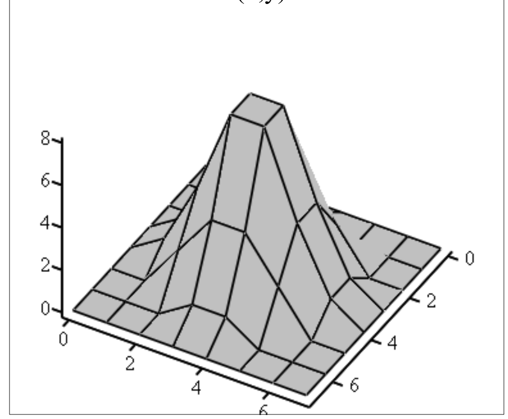
$$k0 := \begin{pmatrix} a & a \\ a & a \end{pmatrix} \qquad k1 := \begin{pmatrix} -a & a \\ -a & a \end{pmatrix}$$

$$k2 := \begin{pmatrix} -a & -a \\ a & a \end{pmatrix} \qquad k3 := \begin{pmatrix} a & -a \\ -a & a \end{pmatrix}$$

Define f(x,y)

$$f \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 4 & 2 & 0 & 0 \\ 0 & 1 & 4 & 8 & 8 & 4 & 1 & 0 \\ 0 & 1 & 4 & 8 & 8 & 4 & 1 & 0 \\ 0 & 0 & 2 & 4 & 4 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

f(x,y)



(1) Convolve f(x,y) with the basis images

$f_{N+1,j} := f_{0,j} \qquad f_{j,N+1} := f_{j,0}$

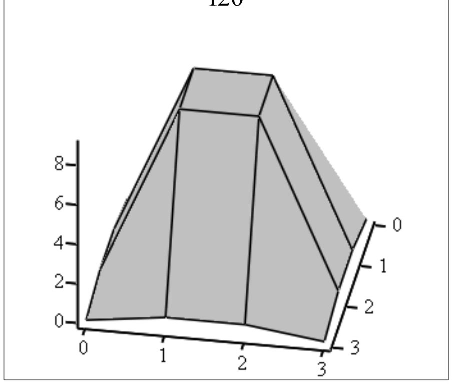
$$f10_{i,j} := \sum_m \sum_n f_{i+m,j+n} \cdot k0_{m,n} \qquad f$$

$$f11_{i,j} := \sum_m \sum_n f_{i+m,j+n} \cdot k1_{m,n} \qquad f12_{i,j} := \sum_m \sum_n f_{i+m,j+n} \cdot k2_{m,n} \qquad f13_{i,j} := \sum_m \sum_n f_{i+m,j+n} \cdot k3_{m,n}$$

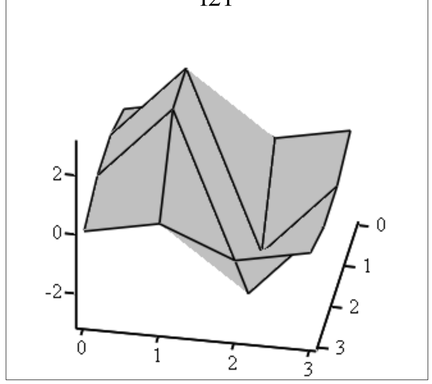
(2) Subsample the rows and columns

$$f20_{i2,j2} := f10_{2\cdot i2,2\cdot j2} \qquad f21_{i2,j2} := f11_{2\cdot i2,2\cdot j2} \qquad f22_{i2,j2} := f12_{2\cdot i2,2\cdot j2} \qquad f23_{i2,j2} := f13_{2\cdot i2,2\cdot j2}$$

f20



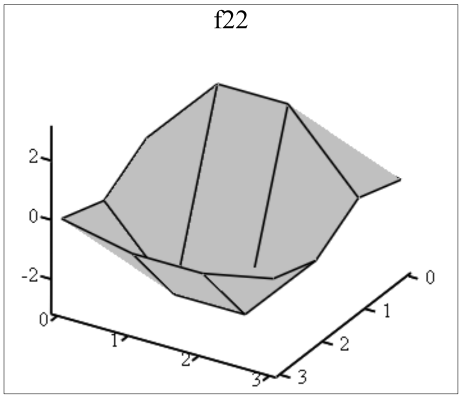
f21



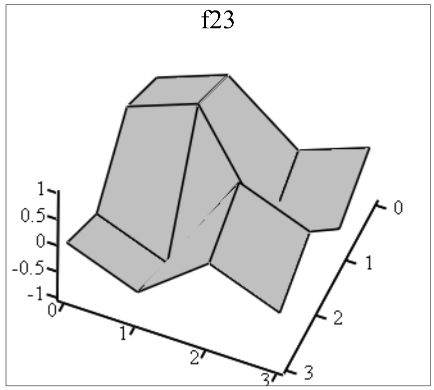
$$f20 = \begin{pmatrix} 0 & 0.5 & 0.5 & 0 \\ 0.5 & 9 & 9 & 0.5 \\ 0.5 & 9 & 9 & 0.5 \\ 0 & 0.5 & 0.5 & 0 \end{pmatrix}$$

$$f21 = \begin{pmatrix} 0 & 0.5 & -0.5 & 0 \\ 0.5 & 3 & -3 & -0.5 \\ 0.5 & 3 & -3 & -0.5 \\ 0 & 0.5 & -0.5 & 0 \end{pmatrix}$$

f22



f23



$$f22 = \begin{pmatrix} 0 & 0.5 & 0.5 & 0 \\ 0.5 & 3 & 3 & 0.5 \\ -0.5 & -3 & -3 & -0.5 \\ 0 & -0.5 & -0.5 & 0 \end{pmatrix}$$

$$f23 = \begin{pmatrix} 0 & 0.5 & -0.5 & 0 \\ 0.5 & 1 & -1 & -0.5 \\ -0.5 & -1 & 1 & 0.5 \\ 0 & -0.5 & 0.5 & 0 \end{pmatrix}$$

f22

f23

NOW THE INVERSE TRANSFORM

Upsample with rows and columns of zeros $\quad i1 := 0..N+1 \quad j1 := 0..N+1$

$g10_{i1,j1} := 0$ $\quad g11_{i1,j1} := 0$ $\quad g12_{i1,j1} := 0$ $\quad g13_{i1,j1} := 0$

$g10_{2\cdot i2+1,\, 2\cdot j2+1} := f20_{i2,j2}$ $\quad g11_{2\cdot i2+1,\, 2\cdot j2+1} := f21_{i2,j2}$ $\quad g12_{2\cdot i2+1,\, 2\cdot j2+1} := f22_{i2,j2}$ $\quad g13_{2\cdot i2+1,\, 2\cdot j2+1} := f23_{i2,j2}$

$$g10 = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&1&0&9&0&9&0&1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&1&0&9&0&9&0&1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \qquad g11 = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&-1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&1&0&3&0&-3&0&-1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&1&0&3&0&-3&0&-1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&-1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$g12 = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&1&0&3&0&3&0&1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&-1&0&-3&0&-3&0&-1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&-1&0&-1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix} \qquad g13 = \begin{pmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&-1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&1&0&1&0&-1&0&-1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&-1&0&-1&0&1&0&1&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&-1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$k0 := \begin{pmatrix} a & a \\ a & a \end{pmatrix} \qquad k1 := -\begin{pmatrix} -a & a \\ -a & a \end{pmatrix} \qquad g0_{i,j} := \sum_m \sum_n g10_{i+m,\, j+n} \cdot k0_{m,n} \qquad g1_{i,j} := \sum_m \sum_n g11_{i+m,\, j+n} \cdot k1_{m,n}$$

$$k2 := -\begin{pmatrix} -a & -a \\ a & a \end{pmatrix} \qquad k3 := \begin{pmatrix} a & -a \\ -a & a \end{pmatrix} \qquad g2_{i,j} := \sum_m \sum_n g12_{i+m,\, j+n} \cdot k2_{m,n} \qquad g3_{i,j} := \sum_m \sum_n g13_{i+m,\, j+n} \cdot k3_{m,n}$$

$$g0 = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&5&5&5&5&0&0 \\ 0&0&5&5&5&5&0&0 \\ 0&0&5&5&5&5&0&0 \\ 0&0&5&5&5&5&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \qquad g1 = \begin{pmatrix} 0&0&-0&0&0&-0&0&0 \\ 0&0&-0&0&0&-0&0&0 \\ -0&0&-2&2&2&-2&0&-0 \\ -0&0&-2&2&2&-2&0&-0 \\ -0&0&-2&2&2&-2&0&-0 \\ -0&0&-2&2&2&-2&0&-0 \\ 0&0&-0&0&0&-0&0&0 \\ 0&0&-0&0&0&-0&0&0 \end{pmatrix}$$

$$g2 = \begin{pmatrix} 0&0&-0&-0&-0&-0&0&0 \\ 0&0&0&0&0&0&0&0 \\ -0&-0&-2&-2&-2&-2&-0&-0 \\ 0&0&2&2&2&2&0&0 \\ 0&0&2&2&2&2&0&0 \\ -0&-0&-2&-2&-2&-2&-0&-0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&-0&-0&-0&-0&0&0 \end{pmatrix}$$

$$g3 = \begin{pmatrix} 0&0&0&-0&-0&0&0&0 \\ 0&0&-0&0&0&-0&0&0 \\ 0&-0&1&-1&-1&1&-0&0 \\ -0&0&-1&1&1&-1&0&-0 \\ -0&0&-1&1&1&-1&0&-0 \\ 0&-0&1&-1&-1&1&-0&0 \\ 0&0&-0&0&0&-0&0&0 \\ 0&0&0&-0&-0&0&0&0 \end{pmatrix}$$

The final result

$$g := (g0 + g1 + g2 + g3) \qquad g = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&0&0&0 \\ 0&0&2&4&4&2&0&0 \\ 0&1&4&8&8&4&1&0 \\ 0&1&4&8&8&4&1&0 \\ 0&0&2&4&4&2&0&0 \\ 0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$