

## Small Kernel from Subsampled MTF

f\_16-13.MCD, 12/19/94

$$\begin{aligned}
 j &:= \sqrt{-1} & N &:= 28 & i &:= 0..N-1 & k &:= 0..N-1 \\
 M &:= 7 & m &:= 0..M-1 & n &:= 0..M-1 & a &:= \text{floor}\left(\frac{M}{2}\right) & b &:= \text{floor}\left(\frac{N}{2}\right) \\
 a &= 3 & b &= 14
 \end{aligned}$$

The 1-D Discrete Fourier Transform computes an M-point transfer function from an M-point impulse response:  
 Suppose the desired Gaussian TF, given as an N-point spectrum vector centered on element b, is

$$\sigma := \frac{N}{12} \qquad G_i := \exp\left[-\frac{(i-b)^2}{2 \cdot \sigma \cdot \sigma}\right] \qquad N = 28 \qquad b = 14$$

$$G^T = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.01 \ 0.04 \ 0.1 \ 0.23 \ 0.44 \ 0.69 \ 0.91 \ 1 \ 0.91 \ 0.69 \ 0.44 \ 0.23 \ 0.1 \ 0.04 \ 0.01 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

whose N-point FIR is:

$$W_{i,k} := \frac{1}{\sqrt{N}} \cdot \exp\left[-j \cdot 2 \cdot \pi \cdot (i-b) \cdot \frac{k-b}{N}\right] \qquad g := \sqrt{\frac{1}{N}} \cdot (W^{-1} \cdot G) \qquad \sum_{i=0}^{N-1} g_i = 1$$

$$g^T = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.01 \ 0.02 \ 0.06 \ 0.12 \ 0.18 \ 0.21 \ 0.18 \ 0.12 \ 0.06 \ 0.02 \ 0.01 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

We set up an M-point TF by subsampling G:  $F := (.0 \ .0 \ .23 \ 1 \ .23 \ .0 \ .0)^T$

Then its M-point FIR centered on element a is

$$M = 7 \qquad a = 3$$

$$f_n := \left(\frac{1}{\sqrt{M}}\right) \cdot \sum_{m=0}^{M-1} F_m \cdot \exp\left[j \cdot 2 \cdot \pi \cdot (n-a) \cdot \frac{m-a}{M}\right] \qquad \sqrt{\frac{1}{M}} \cdot f^T = (0.08 \ 0.13 \ 0.18 \ 0.21 \ 0.18 \ 0.13 \ 0.08)$$

We can transform this back to an N-point spectrum, to see the approximation TF:

If an M-point function is centered on element a, and we want its N-point spectrum centered on element b, we can use:

$$G := T \cdot f \qquad \text{where} \qquad T_{k,m} := \frac{1}{\sqrt{N}} \cdot \exp\left[-j \cdot 2 \cdot \pi \cdot (k-b) \cdot \frac{m-a}{N}\right] \qquad F := \sqrt{\frac{N}{M}} \cdot T \cdot f \qquad \sqrt{\frac{1}{M}} \cdot \sum_{m=0}^{M-1} f_m = 1$$

$$F^T = (-0.07 \ -0.05 \ 0 \ 0.05 \ 0.07 \ 0.05 \ 0 \ -0.05 \ -0.04 \ 0.05 \ 0.23 \ 0.48 \ 0.74 \ 0.93 \ 1 \ 0.93 \ 0.74 \ 0.48 \ 0.23 \ 0.05 \ -0.04 \ -)$$

$$G^T = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.01 \ 0.04 \ 0.1 \ 0.23 \ 0.44 \ 0.69 \ 0.91 \ 1 \ 0.91 \ 0.69 \ 0.44 \ 0.23 \ 0.1 \ 0.04 \ 0.01 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$$

$$\frac{1}{\sqrt{M}} \cdot f^T = (0.08 \ 0.13 \ 0.18 \ 0.21 \ 0.18 \ 0.13 \ 0.08)$$

and the MSE of the approximation is:

$$\text{RMSE} := \sqrt{\frac{1}{N} \cdot \sum_{i=0}^{N-1} (F_i - G_i)^2}$$

$$\text{RMSE} = 0.0484052$$

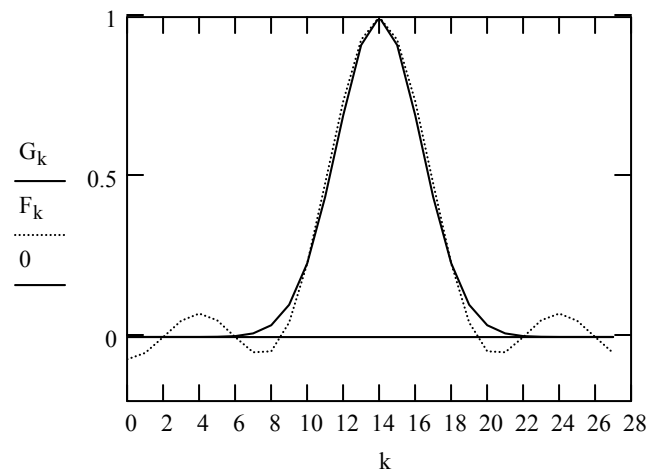


Figure 16-13