Small Kernel from Subsampled MTF

f_16-13.MCD, 12/19/94

$j := \sqrt{-1}$ $\quad N := 28 \quad i := 0..N-1 \quad k := 0..N-1$

$M := 7 \quad m := 0..M-1 \quad n := 0..M-1 \quad a := \mathrm{floor}\left(\frac{M}{2}\right) \quad b := \mathrm{floor}\left(\frac{N}{2}\right)$

$a = 3 \quad b = 14$

The 1-D Discrete Fourier Transform computes an M-point transfer function from an M-point impulse response:

Suppose the desired Gaussian TF, given as an N-point spectrum vector centered on element b, is

$$\sigma := \frac{N}{12} \qquad G_i := \exp\left[-\frac{(i-b)^2}{2\cdot\sigma\cdot\sigma}\right] \qquad N = 28 \qquad b = 14$$

$G^T = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0.01\ \ 0.04\ \ 0.1\ \ 0.23\ \ 0.44\ \ 0.69\ \ 0.91\ \ 1\ \ 0.91\ \ 0.69\ \ 0.44\ \ 0.23\ \ 0.1\ \ 0.04\ \ 0.01\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)$

whose N-point FIR is:

$$W_{i,k} := \frac{1}{\sqrt{N}}\cdot\exp\left[-j\cdot 2\cdot\pi\cdot(i-b)\cdot\frac{k-b}{N}\right] \qquad g := \sqrt{\frac{1}{N}}\cdot\left(W^{-1}\cdot G\right) \qquad \sum_{i=0}^{N-1} g_i = 1$$

$g^T = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0.01\ \ 0.02\ \ 0.06\ \ 0.12\ \ 0.18\ \ 0.21\ \ 0.18\ \ 0.12\ \ 0.06\ \ 0.02\ \ 0.01\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)$

We set up an M-point TF by subsampling G: $\quad F := (.0\ \ .0\ \ .23\ \ 1\ \ .23\ \ .0\ \ .0)^T$

Then its M-point FIR centered on element a is

$M = 7 \qquad a = 3$

$$f_n := \left(\frac{1}{\sqrt{M}}\right)\cdot\sum_{m=0}^{M-1} F_m\cdot\exp\left[j\cdot 2\cdot\pi\cdot(n-a)\cdot\frac{m-a}{M}\right] \qquad \sqrt{\frac{1}{M}}\cdot f^T = (0.08\ \ 0.13\ \ 0.18\ \ 0.21\ \ 0.18\ \ 0.13\ \ 0.08)$$

We can transform this back to an N-point spectrum, to see the approximation TF:
If an M-point function is centered on element a, and we want its N-point spectrum centered on element b, we can use:

$$G := T\cdot f \qquad \text{where} \qquad T_{k,m} := \frac{1}{\sqrt{N}}\cdot\exp\left[-j\cdot 2\cdot\pi\cdot(k-b)\cdot\frac{m-a}{N}\right] \qquad F := \sqrt{\frac{N}{M}}\cdot T\cdot f \qquad \sqrt{\frac{1}{M}}\cdot\sum_{m=0}^{M-1} f_m = 1$$

$F^T = (-0.07\ \ -0.05\ \ 0\ \ 0.05\ \ 0.07\ \ 0.05\ \ 0\ \ -0.05\ \ -0.04\ \ 0.05\ \ 0.23\ \ 0.48\ \ 0.74\ \ 0.93\ \ 1\ \ 0.93\ \ 0.74\ \ 0.48\ \ 0.23\ \ 0.05\ \ -0.04\ \ -$

$G^T = (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0.01\ \ 0.04\ \ 0.1\ \ 0.23\ \ 0.44\ \ 0.69\ \ 0.91\ \ 1\ \ 0.91\ \ 0.69\ \ 0.44\ \ 0.23\ \ 0.1\ \ 0.04\ \ 0.01\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)$

$$\frac{1}{\sqrt{M}}\cdot f^T = (0.08\ \ 0.13\ \ 0.18\ \ 0.21\ \ 0.18\ \ 0.13\ \ 0.08)$$

and the MSE of the approximation is:

$$RMSE := \sqrt{\frac{1}{N}\cdot\sum_{i=0}^{N-1}\left(F_i - G_i\right)^2}$$

$RMSE = 0.0484052$

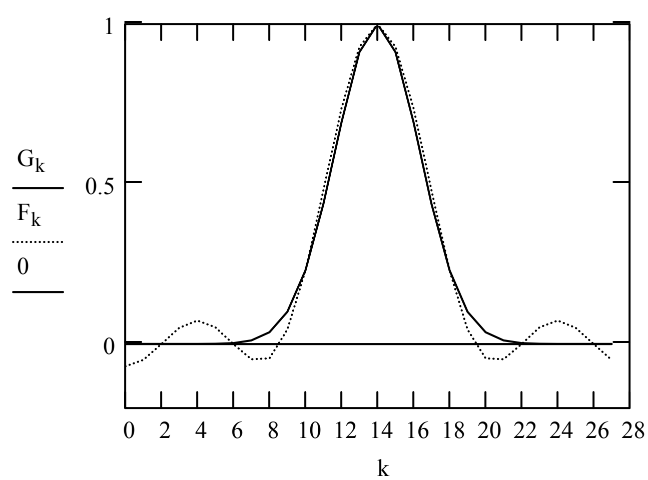

Figure 16-13