$Min := 0 \qquad Max := 10 \qquad Inc := 0.2 \qquad x := Min, Min + Inc .. Max \qquad s := .3 \qquad K := 1$

$i := Min .. Max \qquad \varepsilon \equiv .001 \qquad \sigma := .5 \qquad f(x) := \cos(2 \cdot \pi \cdot s \cdot x)$

$j := 0 .. 3 \cdot Max + 16 \qquad y := Min, Min + Inc .. 3 \cdot Max$

$$S(x,\tau,\sigma) := \text{if}\left[\left(|x-\tau| < 2.5\right), K \cdot e^{-.5 \cdot \left(\frac{x-\tau}{\sigma}\right)^2}, 0\right] \qquad K := \frac{1}{S(0,0,\sigma) + 2 \cdot S(0,1,\sigma)} \qquad K = 0.787$$

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3X Sinc Resampling with Gaussian Interpolation

Ken Castleman
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f_03-12.MCD

Suppose we have a signal $f(x) := \cos(2 \cdot \pi \cdot s \cdot x)$

$s = 0.3$

which is sampled $A_i := f(i)$

and interpolated by

$$S(x,\tau,\sigma) := K \cdot e^{-.5 \cdot \left(\frac{x-\tau}{\sigma}\right)^2}$$

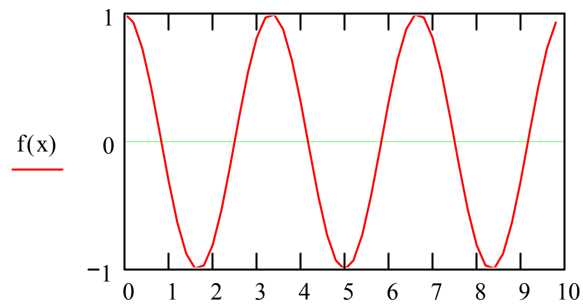
$K = 0.787 \qquad \sigma = 0.5$

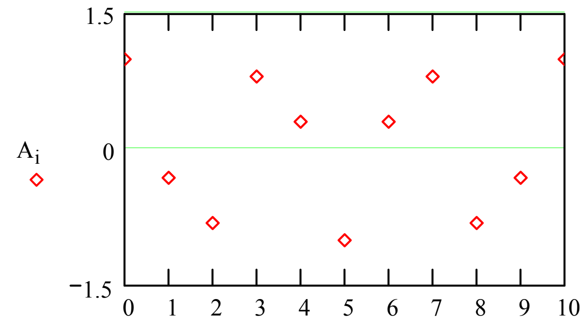
$h(i,x) := A_i \cdot S(x,i,\sigma)$

The interpolated function is

$$g(x) := \sum_i h(i,x)$$

which differs somewhat from the origonal function in both amplitude and waveshape.





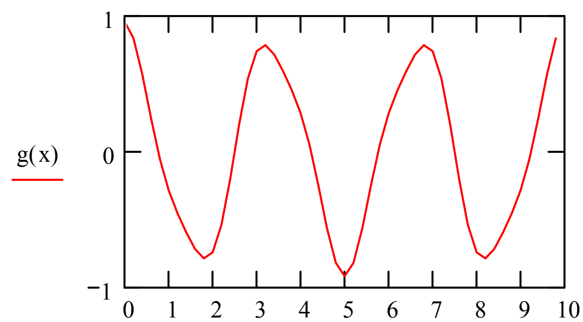


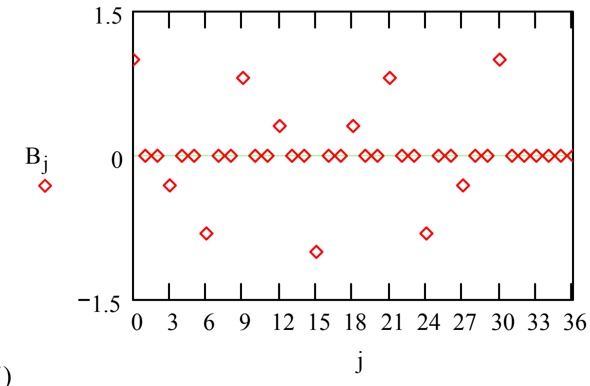
Figure 3-11

Even though the function is oversampled (s = 0.3 cycles per sample), interpolation with a Gaussian interpolation function (display spot) does not give a good reconstruction.

3X Resampling

We can improve the interpolation process by resampling the function at a 3X higher sampling density. The hard way to do this is to perform the origonal sampling at a higher sampling frequency. An easier way is to create two new sample points between each pair of existing sample points by interpolating them with sin(x)/x. Since the function is oversampled to begin with, the result is the same.

$B_j := 0 \qquad B_{3 \cdot i} := A_i$



$$p(x) := \frac{\sin(\pi \cdot x)}{\pi \cdot x}$$

$p(.333) = 0.827$

$p(.667) = 0.413$

$p(1.333) = -0.207$

$p(1.667) = -0.165$

The interpolation coefficients must be normalized to sum to 1.0 so that areas of constant amplitude will be preserved.

$$Sum := p(.333) + p(1.33) + p(2.33) + p(.667) + p(1.667) + p(2.667)$$

$$C1 := \frac{p(2.333)}{Sum} \qquad C2 := \frac{p(1.333)}{Sum} \qquad C3 := \frac{p(0.333)}{Sum} \qquad Sum = 1.09$$

$$C4 := \frac{p(.667)}{Sum} \qquad C5 := \frac{p(1.667)}{Sum} \qquad C6 := \frac{p(2.667)}{Sum}$$

Interpolation fills in the new "synthetic" sample points.

$k := 7, 10 .. 31$

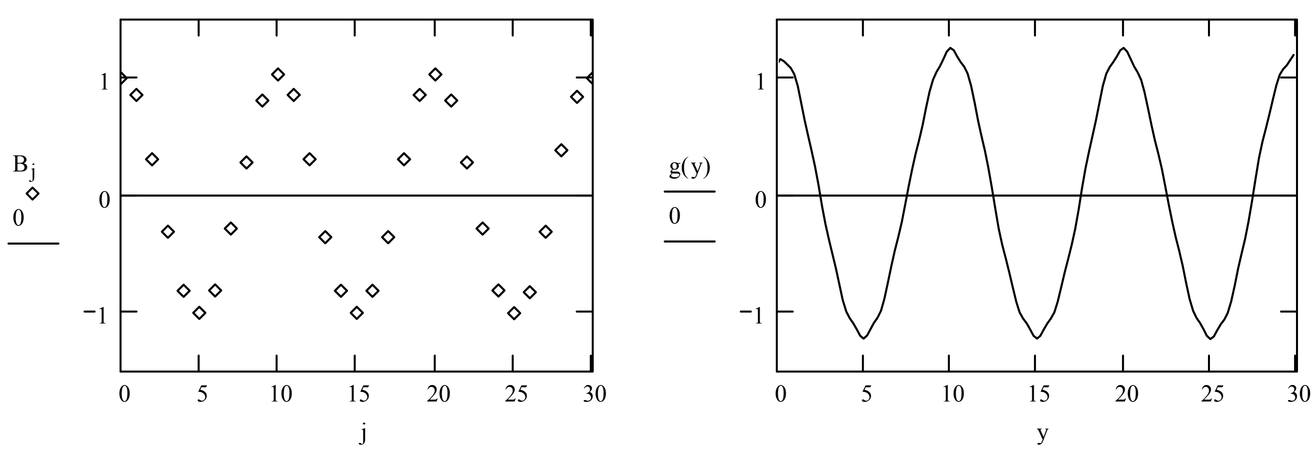
$$B_k := C1 \cdot B_{k-7} + C2 \cdot B_{k-4} + C3 \cdot B_{k-1} + C4 \cdot B_{k+2} + C5 \cdot B_{k+5} + C6 \cdot B_{k+8}$$

$k := 8, 11 .. 32$

$$B_k := C6 \cdot B_{k-8} + C5 \cdot B_{k-5} + C4 \cdot B_{k-2} + C3 \cdot B_{k+1} + C2 \cdot B_{k+4} + C1 \cdot B_{k+7}$$

$B_4 := B_{14} \qquad B_5 := B_{15} \qquad B_1 := B_{11} \qquad B_2 := B_{12}$

$$h(j, y) := B_j \cdot S(y, j, \sigma) \qquad g(y) := \sum_j h(j, y)$$



{a} (b)

Figure 3-12

(a) Notice how this sampled function looks easier to interpolate.
(b) Notice how this interpolated function more closely resembles the origonal waveshape.