# Computing Centered 2-D Fourier Transforms

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DFT_10.MCD, 12/28/95

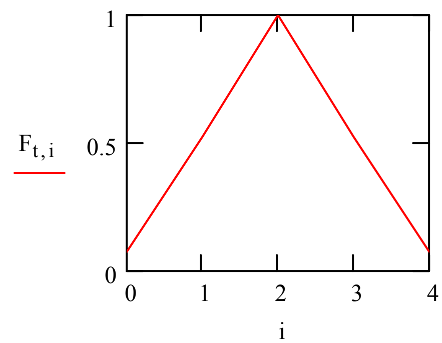
$j := \sqrt{-1}$ $\quad N := 5$ $\quad i := 0 .. N-1$ $\quad k := 0 .. N-1$ $\quad t := \mathrm{floor}\left(\frac{N}{2}\right)$ $\quad t = 2$ $\quad \sigma := 0.88$ $\quad G(x,\sigma) := \exp\left(-\frac{x \cdot x}{2 \cdot \sigma \cdot \sigma}\right)$

The DFT will compute an MTF from an FIR, but it requires an awkward arrangement of the elements in the FIR and MT matrices. The centered DFT formulation used here allows the functions to be centered in their matrices.

Suppose the desired MTF, given as a 5x5 spectrum matrix is

$$F_{i,k} := G\left[\sqrt{(i-t)^2 + (k-t)^2}, \sigma\right]$$

$$F = \begin{pmatrix} 0.01 & 0.04 & 0.08 & 0.04 & 0.01 \\ 0.04 & 0.27 & 0.52 & 0.27 & 0.04 \\ 0.08 & 0.52 & 1 & 0.52 & 0.08 \\ 0.04 & 0.27 & 0.52 & 0.27 & 0.04 \\ 0.01 & 0.04 & 0.08 & 0.04 & 0.01 \end{pmatrix}$$




The 2-D inverse centered DFT computes an NxN-point convolution kernel from an NxN-point transfer function. With this formulation, both the FIR and the MTF are centered in their matrices.

$$W_{i,k} := \frac{1}{N} \cdot \exp\left[j \cdot 2 \cdot \pi \cdot (i-t) \cdot \frac{k-t}{N}\right] \qquad f := (W \cdot F \cdot W)$$

The 5x5 kernel is:

$$f = \begin{pmatrix} 0 & 0.01 & 0.02 & 0.01 & 0 \\ 0.01 & 0.06 & 0.11 & 0.06 & 0.01 \\ 0.02 & 0.11 & 0.19 & 0.11 & 0.02 \\ 0.01 & 0.06 & 0.11 & 0.06 & 0.01 \\ 0 & 0.01 & 0.02 & 0.01 & 0 \end{pmatrix}$$

Since F(0) = 1, the kernel weights add up to one:

$$\sum_{i=0}^{N-1} \sum_{k=0}^{N-1} f_{i,k} = 1$$

The forward centered DFT computes the MTF from the kernel:

$$T_{i,k} := \exp\left[-j \cdot 2 \cdot \pi \cdot (i-t) \cdot \frac{k-t}{N}\right] \qquad F := (T \cdot f \cdot T)$$

$$F = \begin{pmatrix} 0.01 & 0.04 & 0.08 & 0.04 & 0.01 \\ 0.04 & 0.27 & 0.52 & 0.27 & 0.04 \\ 0.08 & 0.52 & 1 & 0.52 & 0.08 \\ 0.04 & 0.27 & 0.52 & 0.27 & 0.04 \\ 0.01 & 0.04 & 0.08 & 0.04 & 0.01 \end{pmatrix}$$

DFT - Discrete Fourier Transform
FIR - Finite Impulse Response (kernel)
MTF - Modulation Transfer Function