Portfolio Assignment:

Binomial Probability

Type I, Mathematical Investigation

Due date: Wednesday, April 5, 2006

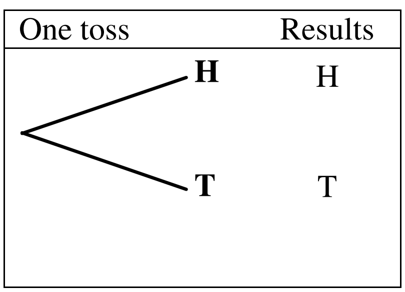
This assignment will be assessed against all six criteria. Justify your answers with good reasoning, which may be in the form of words, tables, graphs, etc. Describe how you use technology to justify a good score in that criterion. Because this assignment is an investigation, you are *not* to research or "look up" any processes. If you have studied this topic in another course, cite that experience when you draw upon it in your explanations.

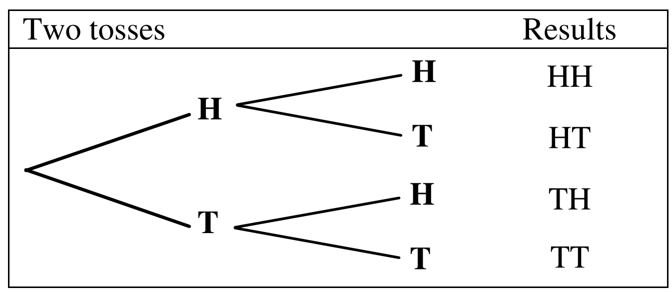
As part of your portfolio, this assignment must represent entirely your own work. You may only discuss it with the teacher; outside of class time, you must work solely on your own. To verify that you have followed these requirements, sign the statement below after completing the assignment, and attach this assignment to the front of your work.

This portfolio assignment represents my own work. I did not seek or receive any unauthorized assistance.

Part 1

The tree diagrams below show the results of tossing a fair coin once and twice, where H represents "heads" and T represents "tails."



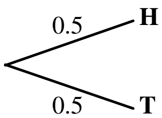
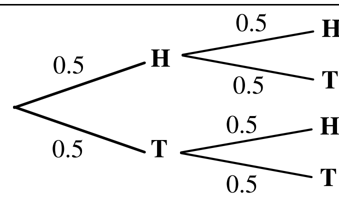


Draw your own trees for three and four flips of a coin. Then copy the table and fill in the first four rows with the number of trials that produce each number of "tails" flips.

number of tosses	no tails	1 tail	2 tails	3 tails	4 tails	5 tails	6 tails	7 tails
1								
2								
3								
4								
5								
6								
7								

By now, you should recognize a pattern in the entries in the table. Describe the pattern you see, and tell where you have encountered it in the past. Based on the pattern, fill in the rest of the blanks in the table.

Part 2
Next, look at the weighted trees for one and two flips of a coin shown here. The probabilities on each branch may be multiplied to find the probability of following that path to an outcome.

One toss	Results	Probability	Two tosses	Results	Probability
	H	0.5		HH	$0.5 \cdot 0.5 = 0.25$
	T	0.5		HT	$0.5 \cdot 0.5 = 0.25$
				TH	$0.5 \cdot 0.5 = 0.25$
				TT	$0.5 \cdot 0.5 = 0.25$

Make your own weighted trees for three and four coin tosses, and make a new table similar to the first, this time with entries that give the probability of each outcome. Based on the pattern you observe, fill in the table up through the seventh row.

In general, without first generating a very large table, how can you find the probability of r tails in n tosses of a fair coin? Explain your reasoning.

Part 3
Suppose that you are faced with a multiple-choice quiz of five questions, each with three choices for the answer. Further suppose that you had intended to study for it this morning but instead overslept, barely slipping into class in time to avoid the dreaded lunch detention.

With three possible answers for each question, if you select an answer at random, the probability that you are correct for a single question is 1/3. Draw a weighted tree to illustrate the situation. (Note that for each answer, although there are three possible choices, there are only two possible outcomes: you are either correct or incorrect. You only need two branches at each step.) Be sure to plan carefully so that you have room to make the tree neat.

Now construct a table and fill in the appropriate probabilities. Your table should include probabilities for a one-question quiz, a two-question quiz, and so on up to five, like the tables in parts 1 and 2. Use your results to state the probability that you will pass the five-question quiz (earn a score of 60% or greater) by guessing at random.

Do you think that the result you have just found is the same as your probability of passing if there are 10 questions instead? First give your intuitive guess, and then show the computations that lead to the true probability of passing by guessing at random on a 10-question multiple-choice quiz with three options for each answer. Comment on how well your intuition conformed to the result of the computation.

Definition
A **binomial experiment** is one that has the following characteristics:
1. There are a fixed number of independent trials; and
2. Each trial has exactly two complementary outcomes.

The two outcomes are most commonly referred to as success and failure; because the trials are independent of each other, the probability of success is the same for each trial.

The examples given here have been binomial. In the case of tossing a coin, the two outcomes are H and T, and coin-tossing is one of the classic probabilistic phenomena assumed to be independent in successive trials. In the quiz situation, the two outcomes are correct and incorrect, and the random nature of your selections makes the successive trials (a/k/a guesses) independent of each other.

Part 4

Based on the results of your explorations thus far, summarize the process for determining the probability of obtaining r successes in n trials of a binomial experiment. Also give a rule for determining the probability of obtaining ***at least*** r successes in n trials. Be sure to describe how use of special calculator commands can help in the computations.

In real life, successive trials may not be truly independent, but it is often convenient to pretend that they are. For instance, if you are one of 500 students in the gymnasium who hope to hear their names called to win a $50 prize, the probability of your being picked first is 1 in 500. However, if you do not win on the first draw, your probability of being selected the second time has risen to 1 in 499, since the name of the first winner has been removed from the drawing. Because the probabilities involved, while different, are so close to each other, we still use the techniques of binomial probability to calculate likelihoods. A good rule of thumb to use is that if the original group has been diminished by no more than 10%, the assumption of independence of trials is still relatively safe.

Likewise, statistics in sports like batting averages and free-throw percentages actually change with every successive attempt, but when the numbers of trials are large, one additional success or failure does not change the probability very much, and it is convenient to treat these as binomial experiments as well.

Part 5

Now consider the following question: A football kicker makes, on average, 85% of his point-after-touchdown attempts. In his next ten attempts, which seems more likely — eight successes, or nine? After giving your intuitive answer, show the work that leads to the calculation of the probabilities of both eight and nine successful PATs, assuming that the trials are independent of each other.

Your graphing calculator should have the capability of computing binomial probabilities; the command is "binompdf."

- If you are using a TI-83 or 84, the command is in the DISTR (for Distributions) menu.
- If you are using a TI-89, you must have the Stats/List Editor APP installed. You can check for this in the APPS, Flash Apps menu. If it's not there, you can install it from the CD that came with the calculator or download it at http://education.ti.com/us/product/apps/statsle.html. After it's installed, access the command from the Catalog menu, F3 for commands from APPS.

For both calculators, the syntax is binompdf(n, p, x), where n is the number of trials, p is the probability of success on any given trial, and x is the number of successes. The number of successes can be a list; in this case, using {8, 9} will give the probabilities of 8 and 9 successes, in that order.

Part 6

Using your calculator to help with the experimentation, determine, to three significant figures, the *smallest* value of the probability of success that makes 9 PATs out of 10 more likely than 8. Do you find the result at all surprising? Comment on your intuition versus the result of the computation.

Based on the rule you developed in part 4 and the formula for binomial coefficients that you learned in studying combinatorics, write and solve an equation to determine the value of p that makes 8 and 9 successes equally likely. Generalize, if you can.