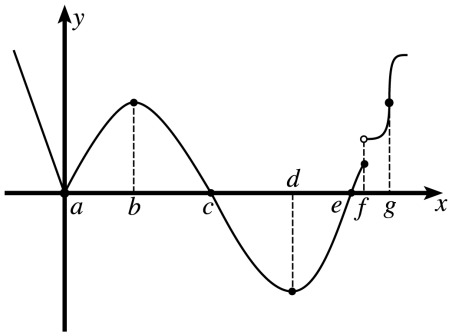
Review for Test, Derivatives

On the test, you will be required to show all of your work. Completion of this worksheet is optional, and it will not be graded. However, the problems are excellent practice for the test, and you are encouraged to ask about any that you do not understand. Please note that there are *far* more problems here than there will be on the test. It has been a long unit, and I wanted you to have many practice problems and examples. The test will count 100 points, and will be divided into calculator and no calculator parts.

1. Let $f(x) = \dfrac{4}{x}$.
   a) Use the *definition* of the derivative to calculate the derivative of f.
   b) Write an equation of the tangent line to the graph of f at the point (–2, –2).
   c) Write an equation of the normal line to the graph for f at the point (–2, –2).

2. Use the graph of $g(x)$ to determine whether the derivative, $g'(x)$, is positive, negative, or zero, or the function has no derivative at each of the values of x listed.
   a) a b) b c) c d) d
   e) e f) f g) g



Differentiate each function with respect to x. Some require implicit differentiation.

3. $y = x\sqrt[3]{x+1}$
4. $f(x) = (2x^2 + 5)^7$
5. $(x^2 + 1)(x^3 + 3)$
6. $y = \left(\dfrac{x}{x^2+1}\right)^{-1}$
7. $r(x) = 3x^2 - \dfrac{2}{x} + \dfrac{5}{x^2}$
8. $y = \dfrac{\pi}{x}$
9. $y = \dfrac{x}{\pi}$
10. $y = 1 + \sqrt{2}\csc x + \cot x$
11. $g(x) = \tan(5 - \sin 2x)$
12. $y = 5\cot\left(\dfrac{2}{x}\right)$
13. $y = \sin(x + y)$
14. $g(x) = \dfrac{1}{\sqrt[4]{6-x^3}}$
15. $y^2 = x$
16. $x^2 - xy + y^2 = 7$
17. $y = x\sqrt{x^2+1}$
18. $y = \sqrt{1-\sqrt{x}}$
19. $y^4 = y^2 - x^2$
20. $x^2\cos^2 y - \sin y = 0$
21. $f(x) = x \ln x$
22. $y = \ln\left(\dfrac{\sqrt{4+x^2}}{x}\right)$
23. $y = \ln\left(\dfrac{1}{x}\right)$
24. $g(x) = \ln\left|\sec x + \tan x\right|$
25. $y = \ln(\sin x)$
26. $y = (\ln x)^5$
27. $g(x) = \arctan x$
28. $h(x) = \operatorname{arccsc}(e^x)$
29. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$
30. $f(x) = \log_a a^{\sin x}$
31. $y = e^{\sqrt{x}}$
32. $y = x^3 3^x$
33. $g(x) = \ln\left(\dfrac{10}{x}\right)$
34. $y = \ln\left(\dfrac{\sqrt{x}}{5-x}\right)$
35. $y = \log_4(4 + x \ln 4)$

36. What is logarithmic differentiation, and why is it used?

Find $\frac{dy}{dx}$ using logarithmic differentiation.

37. $y = \dfrac{\sqrt{3x+1}}{(x-2)\sqrt[4]{x+8}}$ 38. $y = x^{\cos x}$ 39. $y = (x+1)^{x+1}$ 40. $y = x^{\frac{1}{\ln x}}$

41. Find $\dfrac{d^2y}{dx^2}$ for $y = \dfrac{x+2}{x-3}$.

42. Find $\dfrac{d^2y}{dx^2}$ for $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 4$.

43. Let $f(3) = 0$, $f'(3) = 6$, $g(3) = 1$, and $g'(3) = \frac{1}{3}$. Find each of the following, if possible.

a) $h'(3)$ if $h(x) = f(x)g(x)$

b) $j'(3)$, if $j(x) = g(f(x))$

c) $k'(3)$, if $k(x) = \dfrac{g(x)}{h(x)}$

d) $m'(3)$, if $m(x) = 3f(x) - 2g(x)$

44. A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the displacement of the particle from $t = 2$ to $t = 5$.
b) Find the average velocity of the particle from $t = 2$ to $t = 5$.
c) Find a formula for the instantaneous velocity of the particle at any time $t \geq 0$.
d) For what values of t is the velocity of the particle less than zero?
e) Find the acceleration of the particle at any time $t \geq 0$.
f) Find the value of t when the particle is moving and the acceleration is zero.

True or false? Justify your answers.

45. If $f'(7) = 5$, then f is continuous at $x = 7$.

46. If $f'(7) = 5$, then f is decreasing at $x = 7$.

47. If $f'(7) = 5$, then $y - 5 = 5(x - 7)$ is tangent to f at the point where $x = 7$.

48. If $f'(7) = 5$, then $\lim\limits_{\Delta x \to 0} \dfrac{f(5+\Delta x) - f(5)}{\Delta x} = 7$.

49. If $f'(7) = 5$, then $\lim\limits_{x \to 7} \dfrac{f(x) - f(7)}{x - 7} = 5$

50. If $f'(7) = 5$, then $\left(f^{-1}\right)'(7) = \dfrac{1}{5}$.

Evaluate each limit. Use L'Hôpital's Rule if necessary.

51. $\lim\limits_{x \to \pi^-} \dfrac{\sin x}{1 - \cos x}$ 52. $\lim\limits_{x \to -\infty} \dfrac{5x^3 - 2x + 7}{3x^3 + 2x^2}$ 53. $\lim\limits_{x \to 0} \dfrac{\tan x}{x + \sin x}$

54. $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^3}$ 55. $\lim\limits_{x \to 0} \dfrac{\sin x - x}{x^3}$

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