

# AP Calculus AB Free Response Set 1

## For quiz Nov. 16 or Nov. 17

CALCULUS AB  
SECTION II, Part A  
Time—45 minutes  
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. A particle moves along the  $x$ -axis with velocity at time  $t \geq 0$  given by  $v(t) = -1 + e^{1-t}$ .
- (a) Find the acceleration of the particle at time  $t = 3$ .
  - (b) Is the speed of the particle increasing at time  $t = 3$ ? Give a reason for your answer.
  - (c) Find all values of  $t$  at which the particle changes direction. Justify your answer.
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2. The number of gallons,  $P(t)$ , of a pollutant in a lake changes at the rate

$$P'(t) = 1 - 3e^{-0.2\sqrt{t}} \text{ gallons per day,}$$

where  $t$  is measured in days. There are 50 gallons of the pollutant in the lake at time  $t = 0$ . The lake is considered to be safe when it contains 40 gallons or less of pollutant.

- a) Is the amount of pollutant increasing at time  $t = 9$ ? Why or why not?
  - b) For what value of  $t$  will the number of gallons of pollutant be at its minimum? Justify your answer.
  - c) An investigator uses the tangent line approximation to  $P(t)$  at  $t = 0$  as a model for the amount of pollutant in the lake. At what time  $t$  does this model predict that the lake becomes safe?
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3. Consider the curve defined by  $2y^3 + 6x^2y - 12x^2 + 6y = 1$ .

- (a) Show that  $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$ .
  - (b) Write an equation of each horizontal tangent line to the curve.
  - (c) The line through the origin with slope  $-1$  is tangent to the curve at point  $P$ . Find the  $x$ - and  $y$ -coordinates of point  $P$ .
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**CALCULUS AB**  
**SECTION II, Part B**  
**Time—45 minutes**  
**Number of problems—3**

**No calculator is allowed for these problems.**

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4. Consider the curve defined by the equation  $y + \cos y = x + 1$  for  $0 \leq y \leq 2\pi$ .

- (a) Find  $\frac{dy}{dx}$  in terms of  $y$ .
- (b) Write an equation for each vertical tangent to the curve.
- (c) Find  $\frac{d^2y}{dx^2}$  in terms of  $y$ .

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5. A particle moves along the  $x$ -axis so that at any time  $t > 0$  its velocity is given by  $v(t) = t \ln t - t$ .

- (a) Write an expression for the acceleration of the particle.
- (b) For what values of  $t$  is the particle moving to the right?
- (c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

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6. Let  $g$  be the function defined as follows.

$$g(x) = \begin{cases} |x-1| + 3, & x < 1 \\ ax^2 + bx, & x \geq 1 \end{cases}, \text{ where } a \text{ and } b \text{ are constants.}$$

- a) If  $a = 3$  and  $b = 4$ , is  $g$  continuous for all  $x$ ? Justify your answer.
  - b) Describe all values of  $a$  and  $b$  for which  $g$  is a continuous function.
  - c) For what values of  $a$  and  $b$  is  $g$  both continuous and differentiable?
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