Limits

Analysis of graphs

Interplay between geometric and analytic information
Use of calculus to predict and explain observed local and global behavior of a function

Limits of functions (including one-sided limits)

An intuitive understanding of the limiting process
Calculating limits using algebra
Estimating limits from graphs or tables of data

Asymptotic and unbounded behavior

Understanding asymptotes in terms of graphical behavior
Describing asymptotic behavior in terms of limits involving infinity
Comparing relative magnitudes of functions and their rates of change (e. g., contrasting exponential growth, polynomial growth, and logarithmic growth)

Continuity as a property of functions

An intuitive understanding of continuity (close values of the domain lead to close values of the range)
Understanding continuity in terms of limits
Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

Derivatives

Concept of the derivative

Derivative presented geometrically, numerically, and analytically
Derivative interpreted as an instantaneous rate of change
Derivative defined as the limit of the difference quotient
Relationship between differentiability and continuity

Derivative at a point

Slope of a curve at a point, including vertical, nonexistent
Tangent line to a curve at a point; local linear approximation
Instantaneous rate of change as the limit of average rate of change
Approximate rate of change from graphs and tables of values

Derivative as a function

Corresponding characteristics of graphs of f and f'
Relationship between increasing and decreasing behavior of f and the sign of f'
The Mean Value Theorem and its geometric consequences
Equations involving derivatives; translating words to symbols

Second derivatives

Corresponding characteristics of the graphs of f, f', and f''
Relationship between the concavity of f and the sign of f''
Points of inflection as places where concavity changes

Applications of derivatives

- Analysis of curves, including monotonicity and concavity
- Optimization, both absolute (global) and relative (local) extrema
- Modeling rates of change, including related rates problems
- Use of implicit differentiation to find the derivative of an inverse
- Interpretation of derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

Computation of derivatives

- Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trig
- Basic rules for derivatives of sums, products, and quotients
- Chain rule and implicit differentiation

Integrals

Interpretations and properties of definite integrals

- Computation of Riemann sums using left, right, midpoint values
- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

- Basic properties of definite integrals (incl. additivity, linearity)

Applications of integrals

- Knowledge, techniques adapted to many applications
- Emphasis is on using integral of a rate of change to give accumulated change, setting up approximating Riemann sum and representing its limit as a definite integral
- Specific applications include finding the area of a region, volume of a solid with known cross sections, average value of a function, and distance traveled by a particle along a line

Fundamental Theorem of Calculus

- Use of the Fundamental Theorem to evaluate definite integrals
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis

Techniques of antidifferentiation

- Antiderivatives following from derivatives of basic functions
- Antiderivatives by substitution of variables, incl. change of limits for definite integrals

Applications of antidifferentiation

- Finding specific antiderivatives using initial conditions, incl. applications to motion along a line
- Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth

Numerical approximations to definite integrals

- Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values